Chapter 11
Analysis of Variance

Chapter Goals
After completing this chapter, you should be able to:
- Recognize situations in which to use analysis of variance
- Understand different analysis of variance designs
- Perform a single-factor hypothesis test and interpret results
- Conduct and interpret post-analysis of variance pairwise comparisons procedures
- Set up and perform randomized blocks analysis
- Analyze two-factor analysis of variance test with replications results

Chapter Overview
Analysis of Variance (ANOVA)
- One-Way ANOVA
  - F-test
  - Tukey-Kramer test
- Randomized Complete Block ANOVA
  - F-test
  - Fisher's Least Significant Difference test
- Two-factor ANOVA with replication
General ANOVA Setting

- Investigator controls one or more independent variables
  - Called factors (or treatment variables)
  - Each factor contains two or more levels (or categories/classifications)
- Observe effects on dependent variable
  - Response to levels of independent variable
- Experimental design: the plan used to test hypothesis

One-Way Analysis of Variance

- Evaluate the difference among the means of three or more populations
  - Examples: Accident rates for 1st, 2nd, and 3rd shift
  - Expected mileage for five brands of tires
- Assumptions
  - Populations are normally distributed
  - Populations have equal variances
  - Samples are randomly and independently drawn

Completely Randomized Design

- Experimental units (subjects) are assigned randomly to treatments
- Only one factor or independent variable
  - With two or more treatment levels
- Analyzed by
  - One-factor analysis of variance (one-way ANOVA)
- Called a Balanced Design if all factor levels have equal sample size
Hypotheses of One-Way ANOVA

- \( H_0 \): All population means are equal
  - i.e., no treatment effect (no variation in means among groups)
- \( H_a \): Not all of the population means are the same
  - At least one population mean is different
  - i.e., there is a treatment effect
  - Does not mean that all population means are different (some pairs may be the same)

One-Factor ANOVA

- \( H_0 \): All means are the same
  - The Null Hypothesis is True
  - (No Treatment Effect)
  - \( \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \)

- \( H_a \): Not all \( \mu_i \) are the same
  - At least one mean is different
  - The Null Hypothesis is NOT true
  - (Treatment Effect is present)
  - \( \mu_1 = \mu_2 \neq \mu_3 \) or \( \mu_1 \neq \mu_2 \neq \mu_3 \)
Partitioning the Variation

1. Total variation can be split into two parts:

\[ \text{SST} = \text{SSB} + \text{SSW} \]

- SST = Total Sum of Squares
- SSB = Sum of Squares Between
- SSW = Sum of Squares Within

Partitioning the Variation (continued)

Total Variation = the aggregate dispersion of the individual data values across the various factor levels (SST)
- Between-Sample Variation = dispersion among the factor sample means (SSB)
- Within-Sample Variation = dispersion that exists among the data values within a particular factor level (SSW)

Partition of Total Variation

\[ \text{Total Variation (SST)} = \text{Variation Due to Factor (SSB)} + \text{Variation Due to Random Sampling (SSW)} \]

Commonly referred to as:
- Sum of Squares Between
- Sum of Squares Among
- Sum of Squares Explained
- Among Groups Variation

Commonly referred to as:
- Sum of Squares Within
- Sum of Squares Error
- Sum of Squares Unexplained
- Within Groups Variation
Total Sum of Squares

\[ \text{SST} = \text{SSB} + \text{SSW} \]

\[ \text{SST} = \sum_{i=1}^{k} n_i \left( \bar{x}_i - \bar{x} \right)^2 \]

Where:
- \( \text{SST} \): Total sum of squares
- \( k \): number of populations (levels or treatments)
- \( n_i \): sample size from population \( i \)
- \( x_{ij} \): \( j \)th measurement from population \( i \)
- \( \bar{x} \): grand mean (mean of all data values)

\[ \text{SST} = \text{SSB} + \text{SSW} \]

Total Variation (continued)

\[ \text{SST} = (x_{11} - \bar{x})^2 + (x_{12} - \bar{x})^2 + \ldots + (x_{kn} - \bar{x})^2 \]

Sum of Squares Between

\[ \text{SST} = \frac{\text{SSB}}{\text{SSW}} \]

\[ \text{SSB} = \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2 \]

Where:
- \( \text{SSB} \): Sum of squares between
- \( k \): number of populations
- \( n_i \): sample size from population \( i \)
- \( \bar{x}_i \): sample mean from population \( i \)
- \( \bar{x} \): grand mean (mean of all data values)
### Between-Group Variation

**SSB** = \[ \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2 \]

\[ \text{Mean Square Between} = \frac{SSB}{\text{degrees of freedom}} \]

**MSB** = \[ \frac{SSB}{k - 1} \]

### Between-Group Variation (continued)

\[ SSB = n_1 (\bar{x}_1 - \bar{x})^2 + n_2 (\bar{x}_2 - \bar{x})^2 + ... + n_k (\bar{x}_k - \bar{x})^2 \]

**Sum of Squares Within**

\[ SST = SSB + SSW \]

SSW = \[ \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 \]

Where:

- SSW = Sum of squares within
- \( k \) = number of populations
- \( n_i \) = sample size from population \( i \)
- \( \bar{x}_i \) = sample mean from population \( i \)
- \( x_{ij} \) = \( j \)th measurement from population \( i \)
Within-Group Variation

$$SSW = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2$$

Summing the variation within each group and then adding over all groups

$$\text{MSW} = \frac{SSW}{N-k}$$

Mean Square Within = $SSW$/degrees of freedom

Within-Group Variation (continued)

$$SSW = (x_{11} - \bar{x}_1)^2 + (x_{12} - \bar{x}_2)^2 + \ldots + (x_{kni} - \bar{x}_k)^2$$

One-Way ANOVA Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Samples</td>
<td>SSB</td>
<td>k - 1</td>
<td>$\text{MSB} = \frac{SSB}{k - 1}$</td>
<td>F = $\frac{\text{MSB}}{\text{MSW}}$</td>
</tr>
<tr>
<td>Within Samples</td>
<td>SSW</td>
<td>N - k</td>
<td>$\text{MSW} = \frac{SSW}{N - k}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SST</td>
<td>N - 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$k = \text{number of populations}$

$N = \text{sum of the sample sizes from all populations}$

$df = \text{degrees of freedom}$
One-Factor ANOVA
F Test Statistic

- Test statistic
  \[ F = \frac{MSB}{MSW} \]
  - MSB is mean squares between variances
  - MSW is mean squares within variances
- Degrees of freedom
  - \( df_1 = k - 1 \)
  - \( df_2 = N - k \)
  \( k \) is the number of populations
  \( N \) is the sum of sample sizes from all populations

- Null hypothesis: \( H_0: \mu_1 = \mu_2 = \ldots = \mu_k \)
- Alternative hypothesis: At least two population means are different

Interpreting One-Factor ANOVA
F Statistic

- The F statistic is the ratio of the between estimate of variance and the within estimate of variance
- The ratio must always be positive
- \( df_1 = k - 1 \) will typically be small
- \( df_2 = N - k \) will typically be large
- The ratio should be close to 1 if \( H_0: \mu_1 = \mu_2 = \ldots = \mu_k \) is true
- The ratio will be larger than 1 if \( H_0: \mu_1 = \mu_2 = \ldots = \mu_k \) is false

One-Factor ANOVA
F Test Example

You want to see if three different golf clubs yield different distances. You randomly select five measurements from trials on an automated driving machine for each club. At the .05 significance level, is there a difference in mean distance?

<table>
<thead>
<tr>
<th>Club 1</th>
<th>Club 2</th>
<th>Club 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>254</td>
<td>234</td>
<td>200</td>
</tr>
<tr>
<td>263</td>
<td>218</td>
<td>222</td>
</tr>
<tr>
<td>241</td>
<td>235</td>
<td>197</td>
</tr>
<tr>
<td>237</td>
<td>227</td>
<td>206</td>
</tr>
<tr>
<td>251</td>
<td>216</td>
<td>204</td>
</tr>
</tbody>
</table>
### One-Factor ANOVA Example: Scatter Diagram

- **Scatter Diagram**
- **Distance**
- **Club 1**
  - 254
  - 263
  - 241
  - 237
  - 251
- **Club 2**
  - 234
  - 218
  - 235
  - 227
  - 216
- **Club 3**
  - 200
  - 222
  - 197
  - 206
  - 204
- **Means**
  - Club 1: $\bar{x}_1 = 249.2$
  - Club 2: $\bar{x}_2 = 226.0$
  - Club 3: $\bar{x}_3 = 205.8$
- **Overall Mean**
  - $\bar{x} = 227.0$

### One-Factor ANOVA Example

#### Computations

<table>
<thead>
<tr>
<th>Club 1</th>
<th>Club 2</th>
<th>Club 3</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$n_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>254</td>
<td>234</td>
<td>200</td>
<td>263</td>
<td>218</td>
<td>222</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>241</td>
<td>235</td>
<td>197</td>
<td>237</td>
<td>227</td>
<td>206</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>237</td>
<td>227</td>
<td>204</td>
<td>251</td>
<td>216</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **SSB**
  - $SSB = 5 \cdot (249.2 - 227)^2 + (226 - 227)^2 + (205.8 - 227)^2 = 4716.4$
- **SSW**
  - $SSW = (254 - 249.2)^2 + (263 - 249.2)^2 + \ldots + (204 - 205.8)^2 = 1119.6$

- **MSB**
  - $MSB = \frac{SSB}{k - 1} = 2358.2$
- **MSW**
  - $MSW = \frac{SSW}{N - k} = 93.3$

### Test Statistic:

- $F = \frac{MSB}{MSW} = \frac{2358.2}{93.3} = 25.275$

### Decision:

- **Reject $H_0$ at $\alpha = 0.05$**
- There is evidence that at least one $\mu_i$ differs from the rest.
ANOVA -- Single Factor: Excel Output

EXCEL: tools | data analysis | ANOVA: single factor

| SUMMARY | | | | |
| Groups | Count | Sum | Average | Variance |
| Club 1 | 5 | 1246 | 249.2 | 108.2 |
| Club 2 | 5 | 1138 | 228 | 77.5 |
| Club 3 | 5 | 1029 | 205.8 | 94.2 |

<table>
<thead>
<tr>
<th>ANOVA</th>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>P-value</th>
<th>F crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>4716.4</td>
<td>2</td>
<td>2358.2</td>
<td>25.275</td>
<td>4.99E-05</td>
<td>3.885</td>
<td></td>
</tr>
<tr>
<td>Within Groups</td>
<td>1119.6</td>
<td>12</td>
<td>93.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5836.0</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Tukey-Kramer Procedure

- Tells which population means are significantly different
  - e.g.: \( \mu_1 = \mu_2 \neq \mu_3 \)
  - Done after rejection of equal means in ANOVA
- Allows pair-wise comparisons
  - Compare absolute mean differences with critical range

Tukey-Kramer Critical Range

\[
\text{Critical Range} = q_{a} \sqrt{\frac{MSW}{2 \left( \frac{1}{n_i} + \frac{1}{n_j} \right)}}
\]

where:
\( q_{a} \) = Value from standardized range table
MSW = Mean Square Within
\( n_i \) and \( n_j \) = Sample sizes from populations (levels) i and j
The Tukey-Kramer Procedure: Example

1. Compute absolute mean differences:

<table>
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<td>251</td>
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</tbody>
</table>

2. Find the q value from the table in appendix J with k and N - k degrees of freedom for the desired level of α.

\[ q_\alpha = 3.77 \]

3. Compute Critical Range:

\[
\text{Critical Range} = q_\alpha \times \sqrt{\frac{MSW}{2}} \left( \frac{1}{n_1} + \frac{1}{n_2} \right)
\]

\[
= 3.77 \times \sqrt{\frac{93.3}{2}} \left( \frac{1}{1} + \frac{1}{8} \right) = 16.285
\]

4. Compare:

\[ |\bar{x}_1 - \bar{x}_2| = 23.2 \]
\[ |\bar{x}_1 - \bar{x}_3| = 43.4 \]
\[ |\bar{x}_2 - \bar{x}_3| = 20.2 \]

5. All of the absolute mean differences are greater than critical range. Therefore there is a significant difference between each pair of means at 5% level of significance.

Tukey-Kramer in PHStat
Randomized Complete Block ANOVA

- Like One-Way ANOVA, we test for equal population means (for different factor levels, for example)...
- ...but we want to control for possible variation from a second factor (with two or more levels)
- Used when more than one factor may influence the value of the dependent variable, but only one is of key interest
- Levels of the secondary factor are called blocks

Partitioning the Variation

- Total variation can now be split into three parts:

\[
\text{SST} = \text{SSB} + \text{SSBL} + \text{SSW}
\]

- SST = Total sum of squares
- SSB = Sum of squares between factor levels
- SSBL = Sum of squares between blocks
- SSW = Sum of squares within levels

Sum of Squares for Blocking

\[
\text{SSBL} = \sum_{j=1}^{b} k(\bar{x}_j - \bar{\bar{x}})^2
\]

Where:
- \(k\) = number of levels for this factor
- \(b\) = number of blocks
- \(\bar{x}_j\) = sample mean from the \(j^{th}\) block
- \(\bar{x}\) = grand mean (mean of all data values)
Partitioning the Variation

Total variation can now be split into three parts:

\[ \text{SST} = \text{SSB} + \text{SSBL} + \text{SSW} \]

SST and SSB are computed as they were in One-Way ANOVA

\[ \text{SSW} = \text{SST} - (\text{SSB} + \text{SSBL}) \]

Mean Squares

\[ \text{MSBL} = \frac{\text{SSBL}}{b - 1} \]

\[ \text{MSB} = \frac{\text{SSB}}{k - 1} \]

\[ \text{MSW} = \frac{\text{SSW}}{(k - 1)(b - 1)} \]

Randomized Block ANOVA Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Blocks</td>
<td>SSB</td>
<td>b - 1</td>
<td>MSBL</td>
<td>MSBL/MSW</td>
</tr>
<tr>
<td>Between Samples</td>
<td>SSB</td>
<td>k - 1</td>
<td>MSB</td>
<td>MSB/MSW</td>
</tr>
<tr>
<td>Within Samples</td>
<td>SSW</td>
<td>(k-1)(b-1)</td>
<td>MSW</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SST</td>
<td>N - 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

k = number of populations
b = number of blocks
N = sum of the sample sizes from all populations
df = degrees of freedom
### Blocking Test

- **H₀**: \( \mu_1 = \mu_2 = \mu_3 = \ldots \)
- **Hₐ**: Not all block means are equal

**F** = \( \frac{MSBL}{MSW} \)

- Blocking test: \( df_1 = b - 1 \)
- \( df_2 = (k - 1)(b - 1) \)

Reject \( H₀ \) if \( F > F_{\alpha} \)

### Main Factor Test

- **H₀**: \( \mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k \)
- **Hₐ**: Not all population means are equal

**F** = \( \frac{MSB}{MSW} \)

- Main Factor test: \( df_1 = k - 1 \)
- \( df_2 = (k - 1)(b - 1) \)

Reject \( H₀ \) if \( F > F_{\alpha} \)

### Fisher’s Least Significant Difference Test

- To test which population means are significantly different
  - e.g.: \( \mu_1 = \mu_2 \neq \mu_3 \)
  - Done after rejection of equal means in randomized block ANOVA design
- Allows pair-wise comparisons
  - Compare absolute mean differences with critical range

![Critical Range Graph]
Fisher's Least Significant Difference (LSD) Test

\[
\text{LSD} = t_{\alpha/2} \sqrt{\frac{\text{MSW}}{b}}
\]

where:
- \( t_{\alpha/2} \) = Upper-tailed value from Student’s \( t \)-distribution for \( \alpha/2 \) and \((k - 1)(n - 1)\) degrees of freedom
- MSW = Mean square within from ANOVA table
- \( b \) = number of blocks
- \( k \) = number of levels of the main factor

Fisher's Least Significant Difference (LSD) Test (continued)

\[
\text{LSD} = t_{\alpha/2} \sqrt{\frac{\text{MSW}}{b}}
\]

Is \( |\bar{x}_1 - \bar{x}_2| \) > LSD?

Compare:
- \( |\bar{x}_1 - \bar{x}_2| \)
- \( |\bar{x}_1 - \bar{x}_3| \)
- \( |\bar{x}_2 - \bar{x}_3| \)

etc...

Two-Way ANOVA

- Examines the effect of
  - Two or more factors of interest on the dependent variable
  - e.g.: Percent carbonation and line speed on soft drink bottling process
  - Interaction between the different levels of these two factors
    - e.g.: Does the effect of one particular percentage of carbonation depend on which level the line speed is set?
Two-Way ANOVA

Assumptions

- Populations are normally distributed
- Populations have equal variances
- Independent random samples are drawn

Two-Factors of interest: A and B

- a = number of levels of factor A
- b = number of levels of factor B
- N = total number of observations in all cells

Sources of Variation

- SST = Total Variation
- SSA = Variation due to factor A
- SSb = Variation due to factor B
- SSab = Variation due to interaction between A and B
- SSE = Inherent variation (Error)

Degrees of Freedom:

- a - 1
- b - 1
- (a - 1)(b - 1)
- N - ab

SST = SSA + SSb + SSab + SSE
### Two Factor ANOVA Equations

#### Total Sum of Squares:

\[
\text{SST} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n'} (x_{ijk} - \bar{x})^2
\]

#### Sum of Squares Factor A:

\[
\text{SS}_A = bn' \sum_{i=1}^{a} (\bar{x}_i - \bar{x})^2
\]

#### Sum of Squares Factor B:

\[
\text{SS}_B = an' \sum_{j=1}^{b} (\bar{x}_j - \bar{x})^2
\]

#### Sum of Squares Interaction Between A and B:

\[
\text{SS}_{AB} = n' \sum_{i=1}^{a} \sum_{j=1}^{b} (\bar{x}_{ij} - \bar{x}_i - \bar{x}_j + \bar{x})^2
\]

#### Sum of Squares Error:

\[
\text{SSE} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n'} (x_{ijk} - \bar{x}_{ijk})^2
\]

---

**Where:**

- \( a \) = number of levels of factor A
- \( b \) = number of levels of factor B
- \( n' \) = number of replications in each cell

**Variables:**

- \( \bar{x} \) = Grand Mean
- \( \bar{x}_{ij} \) = Mean of each level of factor A
- \( \bar{x}_j \) = Mean of each level of factor B
- \( \bar{x}_{ijk} \) = Mean of each cell
### Mean Square Calculations

- Mean square factor A, \( MS_A \) = \( \frac{SS_A}{a - 1} \)
- Mean square factor B, \( MS_B \) = \( \frac{SS_B}{b - 1} \)
- Mean square interaction, \( MS_{AB} \) = \( \frac{SS_{AB}}{(a - 1)(b - 1)} \)
- Mean square error, \( MSE \) = \( \frac{SSE}{N - ab} \)

### Two-Way ANOVA: The F Test Statistic

- **F Test for Factor A Main Effect**
  - \( H_0: \mu_{A1} = \mu_{A2} = \mu_{A3} = \cdots \)
  - \( H_A: \) Not all \( \mu_{Ai} \) are equal
  - \[ F = \frac{MS_A}{MSE} \]
  - Reject \( H_0 \) if \( F > F_{\alpha} \)

- **F Test for Factor B Main Effect**
  - \( H_0: \mu_{B1} = \mu_{B2} = \mu_{B3} = \cdots \)
  - \( H_A: \) Not all \( \mu_{Bi} \) are equal
  - \[ F = \frac{MS_B}{MSE} \]
  - Reject \( H_0 \) if \( F > F_{\alpha} \)

- **F Test for Interaction Effect**
  - \( H_0: \) factors A and B do not interact to affect the mean response
  - \( H_A: \) factors A and B do interact
  - \[ F = \frac{MS_{AB}}{MSE} \]
  - Reject \( H_0 \) if \( F > F_{\alpha} \)

### Two-Way ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Squares</th>
<th>F Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor A</td>
<td>SS_A</td>
<td>( a - 1 )</td>
<td>( MS_A = \frac{SS_A}{a - 1} )</td>
<td>( \frac{MS_A}{MSE} )</td>
</tr>
<tr>
<td>Factor B</td>
<td>SS_B</td>
<td>( b - 1 )</td>
<td>( MS_B = \frac{SS_B}{b - 1} )</td>
<td>( \frac{MS_B}{MSE} )</td>
</tr>
<tr>
<td>AB (Interaction)</td>
<td>SS_{AB}</td>
<td>( (a - 1)(b - 1) )</td>
<td>( MS_{AB} = \frac{SS_{AB}}{a - 1)(b - 1)} )</td>
<td>( \frac{MS_{AB}}{MSE} )</td>
</tr>
<tr>
<td>Error</td>
<td>SSE</td>
<td>( N - ab )</td>
<td>MSE = ( \frac{SSE}{N - ab} )</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>SST</td>
<td>( N - 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 11

Student Lecture Notes

Features of Two-Way ANOVA

- F Test
- Degrees of freedom always add up:
  - \( N-1 = (N-ab) + (a-1) + (b-1) + (a-1)(b-1) \)
  - Total = error + factor A + factor B + interaction
- The denominator of the F Test is always the same but the numerator is different
- The sums of squares always add up:
  - \( SST = SSE + SSA + SSB + SSAB \)
  - Total = error + factor A + factor B + interaction

Examples:
Interaction vs. No Interaction

- No interaction:
  - Interaction is present:

Chapter Summary

- Described one-way analysis of variance
  - The logic of ANOVA
  - ANOVA assumptions
  - F test for difference in \( k \) means
  - The Tukey-Kramer procedure for multiple comparisons
- Described randomized complete block designs
  - F test
  - Fisher's least significant difference test for multiple comparisons
- Described two-way analysis of variance
  - Examined effects of multiple factors and interaction