CHAPTER FIVE

NUMBER THEORY AND THE REAL NUMBER SYSTEM

Exercise Set 5.1
1. Number theory is the study of numbers and their properties.
2. If a and b are factors of c, then c ÷ a is an integer and c ÷ b is an integer.
3. a) $a$ divides $b$ means that $b$ divided by $a$ has a remainder of zero.
   b) $a$ is divisible by $b$ means that $a$ divided by $b$ has a remainder of zero.
4. A prime number is a natural number greater than 1 that has exactly two factors (or divisors), itself and one.
5. A composite number is a natural number that is divisible by a number other than itself and 1. Any natural number that is not prime is composite.
6. Every composite number can be expressed as a unique product of prime numbers.
7. a) The least common multiple (LCM) of a set of natural numbers is the smallest natural number that is divisible (without remainder) by each element of the set.
   b) Determine the prime factorization of each number. Then find the product of the prime factors with the largest exponent in each of the prime factorizations.
8. a) The greatest common divisor (GCD) of a set of natural numbers is the largest natural number that divides (without remainder) every number in that set.
   b) Determine the prime factorization of each number. Then find the product of the prime factors with the smallest exponent that appears in each of the prime factorizations.
   c) 
   \[
   \begin{array}{c|c}
   2 & 16 \\
   2 & 8 \\
   2 & 4 \\
   2 & \\
   \hline
   16 = 2^4 \\
   \end{array}
   \quad
   \begin{array}{c|c}
   5 & 40 \\
   2 & 8 \\
   2 & 4 \\
   2 & \\
   \hline
   40 = 2^3 \cdot 5 \\
   \end{array}
   \]

   The prime factors with the smallest exponents that appear in each of the factorizations are $2^3$.
   The GCD of 16 and 40 is $2^3 = 8$.
9. Mersenne Primes are prime numbers of the form $2^n - 1$ where $n$ is a prime number.
10. A conjecture is a supposition that has not been proved nor disproved.
11. Goldbach’s conjecture states that every even number greater than or equal to 4 can be represented as the sum of two (not necessarily distinct) prime numbers.
12. Twin primes are of the form $p$, $p+2$, where $p$ is a prime number. An example is 5 & 7.
13. The prime numbers between 1 and 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 91, 97.


15. True; since $54 \div 9 = 6$

16. True; since $36 \div 4 = 9$

17. False; since 21 is divisible by 7.

18. False; since 35 is a multiple of 5.

19. False; since 56 is divisible by 8.

20. True; since $45 \div 15 = 3$.

21. True; If a number is divisible by 10, then it is also divisible by 5.

22. False; If a number is divisible by 10, then it is also divisible by 5.

23. False; If a number is divisible by 3, then the sum of the number’s digits is divisible by 3.

24. True.

25. True; since $2 \times 3 = 6$.

26. True; since $3 \times 4 = 12$.

27. Divisible by 2, 3, 4, 6, 8 and 9.

28. Divisible by 2, 3, 4, 5, 6, 8, and 10.

29. Divisible by 3 and 5.

30. Divisible by 2, 3, 4, 5, 6, 8, and 10.

31. Divisible by 2, 3, 4, 5, 6, 8, and 10.

32. Divisible by none of the numbers.

33. $2 \times 3 \times 4 \times 5 \times 6 = 720$. (other answers are possible)

34. $3 \times 4 \times 5 \times 9 \times 10 = 5400$. (other answers are possible)

35. $5 \left[ \begin{array}{c} 45 \\ 3 \\ 9 \\ \hline 45 = 3^2 \times 5 \end{array} \right]$

36. $2 \left[ \begin{array}{c} 52 \\ 2 \\ 26 \\ \hline 52 = 2^2 \times 13 \end{array} \right]$

37. $2 \left[ \begin{array}{c} 196 \\ 2 \\ 98 \\ \hline 196 = 2^2 \times 7^2 \end{array} \right]$

38. $2 \left[ \begin{array}{c} 198 \\ 3 \\ 99 \\ 3 \\ 33 \\ \hline 198 = 2 \times 3^2 \times 11 \end{array} \right]$

39. $3 \left[ \begin{array}{c} 303 \\ 101 \\ \hline 303 = 3 \times 101 \end{array} \right]$

40. $2 \left[ \begin{array}{c} 400 \\ 2 \\ 200 \\ 2 \\ 100 \\ \hline 400 = 2^4 \times 5^2 \end{array} \right]$
41.  
3 513
3 171
3 57
19
513 = 3^3 \cdot 19

42.  
3 663
13 221
17
663 = 3 \cdot 13 \cdot 17

43.  
2 1336
2 668
2 334
167
1336 = 2^3 \cdot 167

44.  
3 2001
23 667
29
2001 = 3 \cdot 23 \cdot 29

45.  
2 3190
5 1595
11 319
3190 = 2 \cdot 5 \cdot 11 \cdot 29

46.  
3 663
13 221
17
663 = 3 \cdot 13 \cdot 17

47.  The prime factors of 15 and 18 are: 6 = 3 \cdot 2, \quad 15 = 3 \cdot 5
a) The common factor is 3, thus, the GCD = 3.
b) The factors with the greatest exponent that appear in either are 2, 3, 5. Thus, the LCM = 2 \cdot 3 \cdot 5 = 30.

48.  The prime factors of 20 and 36 are: 20 = 2^2 \cdot 5 and 36 = 2^2 \cdot 3^2
a) The common factor is 2^2; thus, the GCD = 4.
b) The factors with the greatest exponent that appear in either is 2^2, 3^2; the LCM = 2^2 \cdot 3^2 \cdot 5 = 180.

49.  The prime factors of 24 and 36 are: 24 = 2^3 \cdot 3, 36 = 2^2 \cdot 3^2
a) The common factors are: 2, 3; thus, the GCD = 2 \cdot 3 = 6.
b) The factors with the greatest exponent that appear in either are: 2^3, 3^2; thus, the LCM = 2^3 \cdot 3^2 = 36.

50.  The prime factors of 22 and 231 are: 22 = 2 \cdot 11, 231 = 3 \cdot 7 \cdot 11
a) The common factor is: 11; thus, the GCD = 11.
b) The factors with the greatest exponent that appear in either are: 2, 3, 7, 11; thus, the LCM = 2 \cdot 3 \cdot 7 \cdot 11 = 462

51.  The prime factors of 40 and 900 are: 40 = 2^3 \cdot 5, 900 = 2^2 \cdot 3^2 \cdot 5^2
a) The common factors are: 2^2, 5; thus, the GCD = 2^2 \cdot 5 = 20.
b) The factors with the greatest exponent that appear in either are: 2^3, 3^2, 5^2; thus, the LCM = 2^3 \cdot 3^2 \cdot 5^2 = 1800

52.  The prime factors of 120 and 240 are: 120 = 2^3 \cdot 3 \cdot 5, 240 = 2^4 \cdot 3 \cdot 5
a) The common factors are: 2^3, 3, 5; thus, the GCD = 2^3 \cdot 3 \cdot 5 = 30.
b) The factors with the greatest exponent that appear in either are: 2^4, 3, 5; thus, the LCM = 2^4 \cdot 3 \cdot 5 = 240

53.  The prime factors of 96 and 212 are: 96 = 2^5 \cdot 3, 212 = 2^2 \cdot 53
a) The common factors are: 2^2; thus, the GCD = 2^2 = 4.
b) The factors with the greatest exponent that appear in either are: 2^5, 3, 53; thus, the LCM = 2^5 \cdot 3 \cdot 53 = 5088

54.  The prime factors of 24, 48, and 128 are: 24 = 2^3 \cdot 3, 48 = 2^4 \cdot 3, 128 = 2^7
a) The common factors are: 2^3; thus, the GCD = 2^3 = 8.
b) The factors with the greatest exponent that appear in either are: 2^7, 3; thus, LCM = 2^7 \cdot 3 = 384

55.  The prime factors of 18, 78, and 198 are: 18 = 2 \cdot 3^2, 78 = 2 \cdot 3 \cdot 13, 198 = 2 \cdot 3^2 \cdot 11
a) The common factors are: 2, 3; thus, the GCD = 2 \cdot 3 = 6.
b) The factors with the greatest exponent that appear in either are: 2^3, 3^2, 11; thus, the LCM = 2^3 \cdot 3^2 \cdot 11 = 396

56.  The prime factors of 51 and 153 are: 51 = 3 \cdot 17, 153 = 3 \cdot 53
a) The common factors are: 3; thus, the GCD = 3.
b) The factors with the greatest exponent that appear in either are: 3, 17, 53; thus, the LCM = 3 \cdot 17 \cdot 53 = 2891

57.  Use the list of primes generated in exercise 13. The next two sets of twin primes are: 17, 19, 29, 31.

58.  No. Any other two consecutive natural numbers will include an even number, and even numbers greater than two are composite.
59. (a) 14, 15 Yes; (b) 21, 30 No; (c) 24, 25 Yes; (d) 119, 143 Yes

60. Fermat number = $2^{2^n} + 1$, where $n$ is a natural number. $2^3 + 1 = 5$, $2^{2^2} + 1 = 2^4 + 1 = 17$.
   $2^2 + 1 = 2^3 + 1 = 257$. These numbers are prime.

61. $4 = 2 + 2$, $6 = 3 + 3$, $8 = 3 + 5$, $10 = 3 + 7$, $12 = 5 + 7$, $14 = 7 + 7$, $16 = 3 + 13$, $18 = 5 + 13$, $20 = 3 + 17$

62. Use the formula $2^n - 1$, where $n$ is a prime number. $2^{2} - 1 = 3$, $2^{3} - 1 = 7$, $2^{5} - 1 = 31$, $2^{7} - 1 = 127$, $2^{15} - 1 = 8191$.

63. The gcd of 350 and 140 is 70 dolls.
64. The gcd of 288 and 192 is 96 cars.
65. The gcd of 432 and 360 is 72 cards.
66. The gcd of 150 and 180 is 30 trees.
67. The lcm of 45 and 60 is 180 mins.
68. The lcm of 3500 and 6000 is 42000 miles.

69. The least common multiple of 5 and 6 is 30. Thus, it will be 30 days before they both have the same night off again.
70. The least common multiple of 15 and 18 is 90. Thus, it will be 90 days before he visits both on the same day again.

71. a) The possible committee sizes are: 4, 5, 10, 20, or 25. b) The number of committees possible are:
   25 committees of 4, 20 committees of 5, 10 committees of 10, 5 committees of 20, or 4 committees of 25.

72. a) 5 = $6 - 1$    7 = $6 + 1$    b) Conjecture: Every prime number greater than 11 = $12 - 1$    13 = $12 + 1$ 3 differs by 1 from a multiple of the number 6.    17 = $18 - 1$    19 = $18 + 1$    23 = $24 - 1$ 29 = $30 - 1$    c) The conjecture appears to be correct.

73. A number is divisible by 15 if both 3 and 5 divide the number.
74. A number is divisible by 22 if both 2 and 11 divide the number.

75. 35 + 15 = 2 with rem. = 5. $15 + 5 = 3$ with rem. = 0. Thus, gcd of 35 and 15 is 5.
76. 28 + 16 = 1 with rem. = 12. $16 + 12 = 1$ with rem. = 4. $12 + 4 = 3$ with rem. = 0. Thus, gcd of 28 and 16 is 4.
77. $108 + 36 = 3$ with rem. = 0. $36 + 3 = 12$ with rem. = 0. Thus, gcd of 108 and 36 is 36.

78. 240 + 76 = 3 with rem. = 12. $76 + 12 = 6$ with rem. = 4. $12 + 4 = 3$ with rem. = 0. Thus, gcd of 240 and 76 is 4.
79. 180 + 150 = 1 with rem. = 30. $150 + 30 = 5$ with rem. = 0. Thus, the gcd of 150 and 180 is 30.
80. $560 + 210 = 2$ w/rem. = 140. $210 + 140 = 1$ w/rem. = 70. $140 + 70 = 2$ w/rem. = 0. Thus, gcd of 210 and 560 is 70.

81. The proper factors of 12 are: 1, 2, 3, 4, and 6. 
82. The proper factors of 28 are: 1, 2, 4, 7, and 14. 
83. The proper factors of 496 are: 1,2,4,8,16,31,62,124, and 248. 1+2+4+8+16+31+62+124+248 = 496
   Thus, 496 is a perfect #.

84. The proper factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, and 24. 
   1+2+3+4+6+8+12+16+24 = 76
   Thus, 48 is not a perfect #.
85. a) \(60 = 2^2 \cdot 3 \cdot 5^1\) Adding 1 to each exponent and then multiplying these numbers, we get 
\[(2+1)(1+1)(1+1) = 3 \cdot 2 \cdot 2 = 12\] divisors of 60.

86. No, 2 and 4 are not unique prime factors since \(4 = 2 \cdot 2\). Any number that 4 divides, 2 will also divide, but 8 does not divide all numbers that are divisible by 4. Some examples are: 4, 12, and 20.

87. The sum of the digits will be a number divisible by 3, thus the number is divisible by 6.

88. The sum of the groups which have the same three digits will always be divisible by three.
(i.e. \(d + d + d = 3d\) and \(3|3d\))

89. \(36,018 = (36,000 + 18); 36,000 \div 18 = 2,000\) and \(18 \div 18 = 1\)
Thus, since \(18 \mid 36000\) and \(18 \mid 18\), \(18 \mid 36018\).

90. \(2^2 - 1 = 3, 2^3 - 1 = 7, 2^5 - 1 = 31, 2^7 - 1 = 127\) are prime numbers, but \(2^{11} - 1 = 2,048 - 1 = 2,047\); and since \(23 \cdot 89 = 2,047\), \(2,047\) is not prime.

91. \(8 = 2+3+3, 9 = 3+3+3, 10 = 2+3+5, 11 = 2+2+7, 12 = 2+5+5, 13 = 3+3+7, 14 = 2+5+7, 15 = 3+5+7, 16 = 2+7+7, 17 = 5+5+7, 18 = 2+5+11, 19 = 3+5+11, 20 = 2+7+11\).

92. (a) \(1000 = 3 + 997\); (b) \(2000 = 3 + 1997\); (c) \(3000 = 29 + 2971\)

Exercise Set 5.2
1. Begin at zero, draw an arrow to the value of the first number. From the tip of that arrow draw another arrow by moving a number of spaces equal to the value of the second number. Be sure to move left if the number is negative and move right if the number is positive. The sum of the two numbers is at the tip of the second arrow.

2. \(-n; \) Additive Inverse = that number when added to \(n\) yields the Additive Identity (= 0); \(n + (-n) = 0\).

3. To rewrite a subtraction problem as an addition problem, rewrite the subtraction sign as an addition sign and change the second number to its additive inverse.

4. The product of two numbers with like signs is a positive number, and the product of two numbers with unlike signs is a negative number.

5. The quotient of two numbers with like signs is a positive number, and the quotient of two numbers with unlike signs is a negative number.

6. If we set \(5 + 0 = x\) and we cross multiply, we get the equation \(0x = 5\). Since \(0 \cdot x = 0\), we get \(5 = 0\), which is a false statement, which means that there is no such number \(x\). Therefore, division by 0 is not allowed.

7. \(-6 + 9 = 3\)  
8. \(4 + (-5) = -1\)  
9. \((-7) + 9 = 2\)

10. \((-3) + (-3) = -6\)  
11. \([6+(-11)]+0 = -5+0 = -5\)  
12. \((2+5)+(-4) = 7+(-4) = 3\)

13. \([(-3)+(-4)]+9 = -7+9 = 2\)  
14. \([8+(-3)]+(-2) = [5]+(-2) = 3\)  
15. \([(-23)+(-9)]+11 = [-32]+11 = -21\)

16. \([5+(-13)]+18 = [-8]+18 = 10\)  
17. \(3 - 6 = -3\)  
18. \(-3-7 = -10\)

19. \(-4-6 = -10\)  
20. \(7-(-1) = 8\)  
21. \(-5-(-3) = -5+3 = -2\)

22. \(-4-4 = -4+(-4) = -8\)  
23. \(14 - 20 = 14 + (-20) = -6\)  
24. \(8 - (-3) = 8+3 = 11\)
25. \( [5+(-3)] \cdot -4 = 2 - 4 = 2 + (-4) = -2 \)
26. \( 6 - (8 + 6) = 6 - 14 = 6 + (-14) = -8 \)
27. \( -4 \cdot 5 = -20 \)
28. \( 4(-3) = -12 \)
29. \( (-12)(-12) = 144 \)
30. \( 5(-5) = -25 \)
31. \( [(-8)(-2)] \cdot 6 = 16 \cdot 6 = 96 \)
32. \( (4)(-5)(-6) = (-20)(-6) = 120 \)
33. \( (5 \cdot 6)(-2) = (30)(-2) = -60 \)
34. \( -26 \div (-13) = 2 \)
35. \( -56 \div 8 = -7 \)
36. \( 23 + (-23) = 6 \)
37. \( 56 \div 8 = 7 \)
38. \( 75 \div 15 = 5 \)
39. \( 210 \div 14 = 15 \)
40. \( 20 \div (-2) = 10 \)
41. \( 186 \div 6 = 31 \)
42. \( 144 \div (-3) = -48 \)
43. \( (-900) \div (-4) = 225 \)
44. \( 56 \div -8 = -7 \)
45. \( -75 \div 15 = -5 \)
46. \( -64 \div 16 = -4 \)
47. \( 56 \div -8 = -7 \)
48. \( 75 \div 15 = 5 \)
49. \( 210 \div 14 = 15 \)
50. \( 186 \div 6 = 31 \)
51. \( 144 \div (-3) = -48 \)
52. \( 56 \div -8 = -7 \)
53. \( 75 \div 15 = 5 \)
54. \( 210 \div 14 = 15 \)
55. \( 186 \div 6 = 31 \)
56. \( 144 \div (-3) = -48 \)
57. \( 56 \div -8 = -7 \)
58. \( 75 \div 15 = 5 \)
59. \( 210 \div 14 = 15 \)
60. \( 186 \div 6 = 31 \)
61. \( 56 \div -8 = -7 \)
62. \( 75 \div 15 = 5 \)
63. \( 210 \div 14 = 15 \)
64. \( 186 \div 6 = 31 \)
65. \( 56 \div -8 = -7 \)
66. \( 75 \div 15 = 5 \)
67. \( 210 \div 14 = 15 \)
68. \( 186 \div 6 = 31 \)
69. \( 56 \div -8 = -7 \)
70. \( 75 \div 15 = 5 \)
73. \(0 + 100 - 40 + 90 - 20 + 80 = 210\) pts.

74. \(14,495 \cdot (-282) = 4,149,518\) feet

75. \(842 \cdot (-927) = 842 + 927 = 1,769\) feet

76. \(8 - 5 + 3 + 4 = 3 + 4 = 7\). The Texans did make a first down.

77. a) \(+1 - (-8) = +1 + 8 = 9\).
   There is a 9 hr. time diff.
   b) \(-5 - (-7) = -5 + 7 = 2\).
   There is a 2 hr. time diff.

78. a) \(+1 - (-8) = +1 + 8 = 9\).
   There is a 9 hr. time diff.
   b) \(-5 - (-7) = -5 + 7 = 2\).
   There is a 2 hr. time diff.

79. \(-1 + 2 - 3 + 4 - 5 + 6 - 9 = 1\) (other answers are possible)

80. a) The next 3 pentagonal numbers are 35, 51, and 70. The \(n^{th}\) pentagonal.
   b) The number is obtained by adding the \(n^{th}\) triangular # (see section 1.1) to the \(n^{th}\) square number (see section 1.1) and subtracting \(n\). For example, if \(n = 4\), the \(4^{th}\) triangular number is 10.

81. \(0 + 1 - 2 + 3 + 4 - 5 + 6 - 7 - 8 + 9 = 1\) (other answers are possible)

82. a) \(\frac{4 + 4}{4 + 4} = 1\) 
   b) \(4 \cdot \left(\frac{4 - \frac{4}{4}}{4}\right) = 12\) 
   c) \(\frac{4}{4} \cdot \frac{4}{4} = 15\) 
   d) \(\frac{4 \cdot \frac{4}{4} \cdot \frac{4}{4}}{4} = 16\) 
   e) \(\frac{4}{4} \cdot \frac{4}{4} = 17\)

Exercise Set 5.3

1. Rational numbers is the set of all numbers of the form \(p/q\), where \(p\) and \(q\) are integers, and \(q \neq 0\).

2. a) Multiply and divide the number by the position value of the last nonzero digit to the right of the decimal point.
   b) \(0.397 = \frac{1000 \cdot 0.397}{1000} = \frac{397}{1000}\)

3. a) Divide both the numerator and the denominator by their greatest common divisor.
   b) \(\frac{15}{27} = \frac{5 \div 3}{9 \div 3} = \frac{5}{9}\)

4. Divide the numerator by the denominator. The quotient is the integer part of the mixed number.
   The fraction part of the mixed number is the remainder divided by the divisor.

5. For positive mixed numbers, multiply the denominator of the fraction by the integer preceding it.
   Add this product to the numerator. This sum is the numerator of the improper fraction; the denominator is the same as the denominator of the mixed number. For negative mixed numbers, you can temporarily ignore the negative sign, perform the conversion described above, and then reattach the negative sign.

6. a) The product of two fractions is found by multiplying the numerators and multiplying the denominators.
b) \[
\frac{15}{16} \cdot \frac{24}{25} = \frac{360}{400} = \frac{360}{400} = \frac{9}{10}
\]

7. a) The reciprocal of a number is 1 divided by the number.

b) The reciprocal of -2 is \[
\frac{1}{-2} = -\frac{1}{2}
\]

8. a) To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

b) \[
\frac{4}{15} \div \frac{16}{55} = \frac{4}{15} \cdot \frac{55}{16} = \frac{220}{240} = \frac{220+20}{240+20} = 11
\]

9. a) To add or subtract two fractions with a common denominator, we add or subtract their numerators and keep the common denominator.

b) \[
\frac{11}{36} + \frac{13}{36} + \frac{24}{36} + \frac{24+12}{36} = \frac{2}{3}
\]

c) \[
\frac{37}{48} - \frac{13}{48} = \frac{24}{48} = \frac{24+24}{48} = \frac{1}{2}
\]

10. a) First rewrite each fraction with a common denominator. Then add or subtract the fractions.

b) \[
\frac{5}{12} + \frac{4}{9} = \frac{3 \cdot 5 + 4 \cdot 9}{3 \cdot 12 + 4 \cdot 9} = \frac{15 + 36}{36} = \frac{31}{36}
\]

c) \[
\frac{5}{6} - \frac{2}{15} = \frac{5 \cdot 6 - 2 \cdot 15}{30} = \frac{25 - 4}{30} = \frac{21}{30} = \frac{7}{10}
\]

11. We can multiply a fraction by the number one in the form of c/c (where c is a nonzero integer) and the number will maintain the same value.

12. Yes. \[
\frac{20}{35} = \frac{20+5}{35+5} = \frac{25}{40} = \frac{4}{7}
\]

13. GCD of 14 and 21 is 7.

\[
\frac{14+7}{21} = \frac{21}{35} = \frac{21+7}{35} = \frac{3}{5}
\]

14. GCD of 21 and 35 is 7.

15. GCD of 26 and 91 is 13.

\[
\frac{26}{91} = \frac{26+13}{91+13} = \frac{2}{7}
\]

16. GCD of 36 and 56 is 4.

\[
\frac{36}{56} = \frac{36+4}{56+4} = \frac{9}{14}
\]

17. GCD of 525 and 800 is 25.

\[
\frac{525}{800} = \frac{525+25}{800+25} = \frac{21}{32}
\]

18. GCD of 13 and 221 is 13.

\[
\frac{13}{221} = \frac{13+13}{221+13} = \frac{1}{17}
\]

19. GCD of 112 and 176 is 16.

\[
\frac{112}{176} = \frac{112+16}{176+16} = \frac{128}{192} = \frac{7}{11}
\]

20. GCD of 120 and 135 is 15.

\[
\frac{120}{135} = \frac{120+15}{135+15} = \frac{8}{9}
\]

21. GCD of 45 and 495 is 45.

\[
\frac{45}{495} = \frac{45+45}{495+45} = \frac{1}{11}
\]

22. GCD of 124 and 148 is 4.

\[
\frac{124}{148} = \frac{124+4}{148+4} = \frac{31}{37}
\]

23. \[
\frac{3}{7} = \frac{(3)(7)+4}{7} = \frac{21+4}{7} = \frac{25}{7}
\]

24. \[
\frac{4}{6} = \frac{(4)(6)+5}{6} = \frac{24+5}{6} = \frac{29}{6}
\]

25. \[
\frac{15}{16} = \frac{-(1)(16)+15}{16} = \frac{-16+15}{16} = \frac{-1}{16} = \frac{31}{16}
\]

26. \[
\frac{-7}{5} = \frac{(7)(5)+1}{5} = \frac{35+1}{5} = \frac{36}{5}
\]
27. \[-\frac{4\cdot 15}{16} = -\frac{(4)(16)+15}{16} = -\frac{64+15}{16} = -\frac{79}{16}\]

28. \[\frac{11}{16} = \frac{(11)(16)+9}{16} = \frac{176+9}{16} = \frac{185}{16}\]

29. \[2\frac{1}{8} = \frac{(2)(8)+1}{8} = \frac{16+1}{8} = \frac{17}{8}\]

30. \[2\frac{3}{4} = \frac{(2)(4)+3}{4} = \frac{8+3}{4} = \frac{11}{4}\]

31. \[\frac{7}{8} = \frac{(1)(8)+7}{8} = \frac{8+7}{8} = \frac{15}{8}\]

32. \[\frac{11}{2} = \frac{(1)(2)+1}{2} = \frac{2+1}{2} = \frac{3}{2}\]

33. \[\frac{11}{8} = \frac{8+3}{8} = \frac{(1)(8)+3}{8} = \frac{3}{8}\]

34. \[\frac{23}{4} = \frac{20+3}{4} = \frac{(5)(4)+3}{4} = \frac{5\frac{3}{4}}{4}\]

35. \[-\frac{73}{6} = \frac{-(72+1)}{6} = \frac{-(12\cdot 6+1)}{6} = -\frac{73}{6}\]

36. \[-\frac{457}{11} = \frac{-451+6}{11} = \frac{-(41)(11)+6}{11} = \frac{-(58)(15)+8}{15} = \frac{-58\frac{8}{15}}{15} = \frac{(48)(21)+20}{21} = \frac{48\frac{20}{21}}{21}\]

37. \[\frac{3}{5} = .60\]

38. \[\frac{1028}{21} = \frac{1008+20}{21}\]

39. \[\frac{15}{16} = .9375\]

40. \[\frac{2}{9} = .\overline{2}\]

41. \[5 + 6 = 0.8\overline{3}\]

42. \[3 + 8 = 0.375\]

43. \[23 ÷ 7 = 0.285714\]

44. \[23 ÷ 7 = 4.\overline{3}\]

45. \[13 + 3 = 4.\overline{3}\]

46. \[115 + 15 = 7.\overline{6}\]

47. \[85 ÷ 15 = 5.\overline{6}\]

48. \[1002 ÷ 11 = 91.09\]

49. \[0.25 = \frac{25}{100} = \frac{25+25}{100+25} = \frac{1}{4}\]

50. \[0.29 = \frac{29}{100}\]

51. \[0.045 = \frac{45}{1000} = \frac{45+5}{1000+5} = \frac{9}{200}\]

52. \[0.0125 = \frac{125}{10000} = \frac{1}{80}\]

53. \[0.2 = \frac{2}{10} = \frac{1}{5}\]

54. \[.251 = \frac{251}{1000}\]

55. \[.452 = \frac{452}{1000} = \frac{113}{250}\]

56. \[.2345\]

57. \[.0001 = \frac{1}{10000}\]

58. \[.2535 = \frac{2535}{10000} = \frac{507}{2000}\]

59. \[\text{Let } n = 0.\overline{3}, \quad 10n = 3.\overline{3}\]

60. \[\text{Let } n = 0.\overline{5}, \quad 10n = 5.\overline{5}\]

61. \[\text{Let } n = 1.\overline{9}, \quad 10n = 19.\overline{9}\]
62. Let \( n = 0.51 \), \( 100n = 51.51 \)

\[
\begin{align*}
100n &= 51.51 \\
-n &= -0.51 \\
99n &= 51.0 \\
99 &= 99 = 17 = n
\end{align*}
\]

63. Let \( n = 1.36 \), \( 100n = 136.36 \)

\[
\begin{align*}
100n &= 136.36 \\
-n &= -1.36 \\
99n &= 135.0 \\
99 &= 99 = 15 = n
\end{align*}
\]

64. Let \( n = 1.35 \), \( 1000n = 135.135 \)

\[
\begin{align*}
1000n &= 135.135 \\
-n &= -1.35 \\
999n &= 135.0 \\
999 &= 999 = 5 = n
\end{align*}
\]

65. Let \( n = 1.02 \), \( 100n = 102.2 \)

\[
\begin{align*}
100n &= 102.2 \\
-10n &= -1.02 \\
90n &= 92.0 \\
90 &= 90 = 46 = n
\end{align*}
\]

66. Let \( n = 2.49 \), \( 100n = 249.9 \)

\[
\begin{align*}
100n &= 249.9 \\
-10n &= -2.49 \\
90n &= 225.0 \\
90 &= 90 = 5 = n
\end{align*}
\]

67. Let \( n = 3.478 \)

\[
\begin{align*}
1000n &= 3478.78 \\
10 &= 10 = 34.78 \\
90 &= 90 = 5 = n
\end{align*}
\]

68. Let \( n = 5.239 \), \( 1000n = 5239.39 \)

\[
\begin{align*}
1000n &= 5239.39 \\
-10n &= -52.39 \\
990n &= 5187.0 \\
990 &= 990 = 1729 = n
\end{align*}
\]

69. \[
\frac{4}{11} \cdot \frac{3}{8} = \frac{4 \cdot 3}{11 \cdot 8} = \frac{12}{88} = \frac{12 + 4}{88 + 4} = \frac{3}{22}
\]

70. \[
\frac{3}{5} \cdot \frac{6}{7} = \frac{3 \cdot 6}{5 \cdot 7} = \frac{21}{30} = \frac{21 + 3}{30 + 3} = \frac{7}{10}
\]

71. \[
\frac{-3}{8} \cdot \frac{-16}{15} = \frac{-48}{120} = \frac{2}{5}
\]

72. \[
\left( \frac{-3}{5} \right) + \frac{10}{21} = \left( \frac{-3}{5} \right) \cdot \frac{21}{10} = -\frac{63}{50}
\]

73. \[
\frac{7}{8} + \frac{8}{7} = \frac{7 \cdot 7 + 8 \cdot 8}{8 \cdot 7} = \frac{49}{56} = \frac{49}{64}
\]

74. \[
\frac{3}{7} + \frac{3}{7} = \frac{21}{21} = 1
\]

75. \[
\left( \frac{3}{5} \cdot \frac{4}{7} \right) + \frac{1}{3} = \frac{12}{35} + \frac{1}{3} = \frac{12}{35} \cdot \frac{3}{3} = \frac{36}{35}
\]

76. \[
\left( \frac{4}{7} + \frac{5}{7} \right) \cdot \frac{1}{7} = \frac{4}{7} \cdot \frac{1}{7} + \frac{5}{7} \cdot \frac{1}{7} = \frac{5}{49}
\]

77. \[
\left[ \left( \frac{-3}{4} \right) \left( \frac{-2}{7} \right) \right] \cdot \frac{3}{5} = \left( \frac{6}{28} \right) \cdot \frac{3}{5} = \frac{15}{42} = \frac{5}{14}
\]

78. \[
\left( \frac{3}{7} + \frac{5}{8} \right) \cdot \frac{1}{7} = \left( \frac{15}{72} \right) \cdot \frac{4}{7} \cdot \frac{8}{5} = \frac{5}{24} \cdot \frac{32}{35} = \frac{160}{840} = \frac{4}{21}
\]

79. The lcm of 3 and 5 is 15.

80. The lcm of 6 and 8 is 24.

\[
\frac{5}{6} - \frac{1}{8} = \left( \frac{5}{6} \cdot \frac{4}{4} \right) - \left( \frac{1}{3} \cdot \frac{3}{3} \right) = \frac{20}{24} - \frac{3}{24} = \frac{17}{24}
\]

81. The lcm of 13 and 26 is 26.

\[
\frac{5}{13} + \frac{11}{26} = \left( \frac{5}{13} \cdot \frac{2}{2} \right) + \frac{11}{26} = \frac{10}{26} + \frac{11}{26} = \frac{21}{26}
\]

82. The lcm of 12 and 36 is 36.

\[
\frac{5}{12} + \frac{7}{36} = \left( \frac{5}{12} \cdot \frac{3}{3} \right) + \frac{7}{36} = \frac{15}{36} + \frac{7}{36} = \frac{22}{36} + \frac{2}{36} = \frac{11}{18}
\]

83. The lcm of 9 and 54 is 54.

\[
\frac{5}{9} - \frac{7}{54} = \left( \frac{5}{9} \cdot \frac{6}{6} \right) - \frac{7}{54} = \frac{30}{54} - \frac{7}{54} = \frac{23}{54}
\]
84. The lcm of 30 and 120 is 120.
\[
\begin{align*}
\frac{13}{30} + \frac{17}{120} &= \left(\frac{13}{30} \cdot \frac{4}{4}\right) + \frac{17}{120} = \frac{52}{120} - \frac{17}{120} = \frac{35}{120} \\
\frac{35 + 5}{120 + 5} &= \frac{7}{24}
\end{align*}
\]
85. The lcm of 12, 48, and 72 is 144.
\[
\begin{align*}
\frac{1}{12} + \frac{1}{48} + \frac{1}{72} &= \left(\frac{1}{12} \cdot \frac{3}{3}\right) + \frac{1}{48} + \left(\frac{1}{72} \cdot \frac{2}{2}\right) \\
\frac{12 + 3 + 2}{144} + \frac{1}{144} + \frac{1}{144} &= \frac{17}{144}
\end{align*}
\]
86. The lcm of 5,15, and 75 is 75.
\[
\begin{align*}
\frac{3}{5} + \frac{7}{15} + \frac{9}{75} &= \left(\frac{3}{5} \cdot \frac{15}{15}\right) + \left(\frac{7}{15} \cdot \frac{5}{5}\right) + \frac{9}{75} \\
\frac{45 + 35 + 9}{75 + 75 + 75} &= \frac{89}{225} = \frac{89}{225}
\end{align*}
\]
87. The lcm of 30, 40, and 50 is 600.
\[
\begin{align*}
\frac{1}{30} + \frac{1}{40} + \frac{1}{50} &= \left(\frac{1}{30} \cdot \frac{20}{20}\right) + \frac{1}{40} + \left(\frac{1}{50} \cdot \frac{15}{15}\right) \\
\frac{20 + 45 + 84}{600} &= \frac{109}{600}
\end{align*}
\]
88. The lcm of 25, 100, and 40 is 200.
\[
\begin{align*}
\frac{4}{25} + \frac{9}{100} + \frac{7}{40} &= \left(\frac{4}{25} \cdot \frac{8}{8}\right) + \frac{9}{100} + \left(\frac{7}{40} \cdot \frac{5}{5}\right) \\
\frac{32}{200} + \frac{18}{200} + \frac{35}{200} &= \frac{85}{200}
\end{align*}
\]
89. The lcm of 2, 4, 6 is 12.
\[
\begin{align*}
\frac{1}{2} + \frac{1}{4} + \frac{1}{6} &= \left(\frac{1}{2} \cdot \frac{6}{6}\right) + \frac{1}{4} + \left(\frac{1}{6} \cdot \frac{2}{2}\right) \\
\frac{6 + 3 + 2}{12} + \frac{3}{12} + \frac{2}{12} &= \frac{11}{12}
\end{align*}
\]
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102. \[
\begin{align*}
73 \frac{1}{4} & \rightarrow 73 \frac{5}{4} \\
-69 \frac{3}{4} & \rightarrow -69 \frac{3}{4} \\
3 \frac{2}{4} & \rightarrow 3 \frac{1}{2} \text{ inches}
\end{align*}
\]

105. \[
\begin{align*}
\left( \frac{1}{2} \right) \left( \frac{1}{4} \right) & = \frac{3}{8} \text{ cups of snipped parsley} \\
\left( \frac{1}{2} \right) \left( \frac{1}{8} \right) & = \frac{3}{16} \text{ tsp of pepper} \\
\left( \frac{1}{2} \right) \left( \frac{1}{2} \right) & = \frac{3}{4} \text{ cups of sliced carrots}
\end{align*}
\]

107. The LCM of 4, 5, 3 is 60.
\[
\frac{1}{4} + \frac{2}{5} + \frac{1}{3} = \frac{15}{60} + \frac{24}{60} + \frac{20}{60} = \frac{59}{60}
\]

109. \[
1 - \left( \frac{1}{4} + \frac{1}{5} + \frac{1}{2} \right) = 1 - \left( \frac{5}{20} + \frac{4}{20} + \frac{10}{20} \right) = 1 - \frac{19}{20} = \frac{1}{20}
\]
She must proofread .05 of the book or = 27 pages.

110. \[
\left( \frac{1}{4} \right) \left( 15 \right) = \frac{5}{4} \left( \frac{15}{1} \right) = \frac{75}{4} = 18 \frac{3}{4} \text{ cups}
\]

111. \[
\frac{4}{2} + 30 \frac{1}{4} + 24 \frac{1}{8} = 4 \frac{4}{8} + 30 \frac{2}{8} + 24 \frac{1}{8} = 58 \frac{7}{8} \text{ inches}
\]

113. \[
\left( 24 \frac{7}{8} \right) + 2 = 199 \cdot \frac{1}{2} = 99 \frac{1}{16} = 12 \frac{7}{16} \text{ in.}
\]

115. \[
8 \frac{3}{4} \text{ ft} = \left( \frac{35}{4} \right) \text{ in.} = 105 \text{ in.}
\]
\[
\left[ 105 - (3) \left( \frac{1}{8} \right) \right] + 4 = \frac{840}{8} - \frac{3}{8} + 4 = \frac{837}{4} \cdot \frac{1}{32} = 26 \frac{5}{32}. \text{ The length of each piece is } 26 \frac{5}{32} \text{ in.}
\]

116. original area = \[
8 \frac{1}{2} \cdot 9 \frac{1}{4} = 17 \cdot \frac{37}{4} = 629 \cdot \frac{8}{8} = 78 \frac{5}{8} \text{ sq. in.; new area = } 8 \frac{1}{2} \cdot 10 \frac{1}{4} = 17 \cdot \frac{41}{4} = 697 \cdot \frac{8}{8} = 87 \frac{1}{8} \text{ sq. in.}
\]
area increase = \[
87 \frac{1}{8} - 78 \frac{5}{8} = 86 \frac{9}{8} - 78 \frac{5}{8} = 8 \frac{4}{8} = 8 \frac{1}{2} \text{ sq. in.}
\]
117. width = 8 ft. 3 in. = 96 in. + 3 in. = 99 in.; length = 10 ft. 8 in. = 120 in. + 8 in. = 128 in.
   a) perimeter = 2L + 2W = 2(128) + 2(99) = 454 in.
   b) width = 8 ft. 3 in. = 8 \frac{3}{12} ft. = 8 \frac{1}{4} ft.; length = 10 ft. 8 in. = 10 \frac{8}{12} ft. = 10 \frac{2}{3} ft.

Area = L \times w = \frac{32}{3} \times \frac{33}{4} \times \frac{55}{6} = \frac{58080}{72} = 806.7 \text{ sq. ft.}

117. c) Volume = L \times W \times H = \frac{32}{3} \times \frac{33}{4} \times \frac{55}{6} = \frac{58080}{72} = 806.7 \text{ cu. ft.}

118. a) \[20 + 18 \frac{3}{8} + 2 = 20 + 9 \frac{3}{16} = 29 \frac{3}{16} \text{ in.}\]
   b) \[26 \frac{1}{4} + 6 \frac{3}{4} = 33 \text{ in.}\]
   c) \[26 \frac{1}{4} + \left(6 \frac{3}{4} - \frac{1}{4}\right) = 26 \frac{1}{4} + 6 \frac{2}{4} = 32 \frac{3}{4}\]

121. \[-2.176 + (-2.175) = \frac{-4.351}{2} = -2.1755\]

123. \[3.12345 + 3.123451 = \frac{6.246901}{2} = 3.1234505\]

125. \[4.872 + 4.873 = \frac{9.745}{2} = 4.8725\]

127. \[\left(\frac{1}{3} + \frac{2}{3}\right) = 2 = \frac{3}{1} \times \frac{3}{6} = 2\]

129. \[\left(\frac{1}{100} + \frac{1}{10}\right) = 2 = \frac{11}{100} \times \frac{1}{2} = \frac{11}{200}\]

133. \[\left(\frac{1}{100} + \frac{1}{10}\right) = 2 = \left(\frac{10}{100} + \frac{1}{100}\right) \times \frac{1}{2} = \frac{11}{100} \times \frac{1}{2} = \frac{11}{200}\]

135. a) Water (or milk): \[\left(1 + \frac{3}{4}\right)^2 = \left(\frac{4}{4} + \frac{7}{4}\right) \times \frac{1}{2} = \frac{11}{4} \times \frac{1}{2} = \frac{11}{8} = 1 \frac{3}{8} \text{ cup;}\]
    Oats: \[\left(\frac{1}{2} + 1\right)^2 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4} \text{ cup}\]
136. a) $1$  b) $0.9$  c) $\frac{1}{3} = 0.\overline{3}$, $\frac{2}{3} = 0.\overline{6}$, $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$, $0.\overline{3} + 0.\overline{6} = 1$  d) $0.\overline{9} = 1$

136. a) $\frac{1}{8}$  b) $\frac{1}{16}$  c) 5 times  d) 5 times

Exercise Set 5.4
1. A rational number can be written as a ratio of two integers, $p/q$, with $q$ not equal to zero. Numbers that cannot be written as the ratio of two integers are called irrational numbers.
2. The principal square root of a number $n$ written $\sqrt{n}$, is the positive number that when multiplied by itself gives $n$.
3. A perfect square number is any number that is the square of a natural number.
4. The product rule for radical numbers: $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \quad a, b \geq 0$
   The quotient rule for radical numbers: $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad a, b \geq 0$
5. a) To add or subtract two or more square roots with the same radicand, add or subtract their coefficients and then multiply by the common radical.
   b) $3\sqrt{6} + 5\sqrt{6} - 9\sqrt{6} = 8\sqrt{6} - 9\sqrt{6} = -1\sqrt{6} = -\sqrt{6}$
6. A rationalized denominator contains no radical expressions.
7. a) Multiply both the numerator and denominator by the same number that will result in the radicand in the denominator becoming a perfect square.
   b) $\frac{7}{\sqrt{3}} = \frac{7 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{7\sqrt{3}}{3}$
8. (a) $\sqrt{7}$  [Enter]
   (b) $\sqrt{7} = 2.645751311 = 2.65$
9. $\sqrt{36} = 6$  rational
10. $\sqrt{18} = \sqrt{2 \cdot 9} = 3\sqrt{2}$  irrational
11. $\frac{2}{3}$  rational
12. Irrational; non-terminating, non-repeating decimal
13. Irrational; non-terminating, non-repeating decimal
14. Irrational; $\pi$ is non-terminating, non-repeating.
15. Rational; quotient of two integers
16. Rational; terminating decimal
17. Irrational; non-terminating, non-repeating decimal
18. Rational; $\frac{\sqrt{5}}{\sqrt{5}} = 1$  1 is an integer.
19. $\sqrt{64} = 8$
20. $\sqrt{144} = 12$
21. $\sqrt{100} = 10$
22. $-\sqrt{144} = -12$
23. $-\sqrt{169} = -13$
24. $\sqrt{25} = 5$
25. $-\sqrt{225} = -15$
26. $-\sqrt{36} = -6$
27. $-\sqrt{100} = -10$
28. $\sqrt{256} = 16$
29. 1, rational, integer, natural
30. $-5$, rational, integer
31. $\sqrt{25} = 5$, rat’l, integer., nat’l
32. rational
33. rational
34. rational
35. rational
36. rational

39. $\sqrt{18} = \sqrt{2\cdot9} = 3\sqrt{2}$

40. $\sqrt{20} = \sqrt{4\cdot5} = 2\sqrt{5}$

41. $\sqrt{48} = \sqrt{3\cdot16} = 4\sqrt{3}$

42. $\sqrt{60} = \sqrt{4\cdot15} = 2\sqrt{15}$

43. $\sqrt{63} = \sqrt{9\cdot7} = 3\sqrt{7}$

44. $\sqrt{75} = \sqrt{25\cdot3} = 5\sqrt{3}$

45. $\sqrt{80} = \sqrt{16\cdot5} = 4\sqrt{5}$

46. $\sqrt{90} = \sqrt{9\cdot10} = 3\sqrt{10}$

47. $\sqrt{162} = \sqrt{81\cdot2} = 9\sqrt{2}$

48. $\sqrt{300} = \sqrt{100\cdot3} = 10\sqrt{3}$

49. $2\sqrt{6} + 5\sqrt{6} = (2+5)\sqrt{6} = 7\sqrt{6}$

50. $3\sqrt{17} + \sqrt{17} = (3+1)\sqrt{17} = 4\sqrt{17}$

51.

52.

53. $4\sqrt{12} - 7\sqrt{27} = 4\sqrt{4\cdot3} - 7\sqrt{9\cdot3} = 4\cdot2\sqrt{3} - 7\cdot3\sqrt{3} = 8\sqrt{3} - 21\sqrt{3} = -13\sqrt{3}$

54.

55.

56. $2\sqrt{7} + 5\sqrt{28} = 2\sqrt{7} + 5\cdot2\sqrt{7} = 2\sqrt{7} + 10\sqrt{7} = (2+10)\sqrt{7} = 12\sqrt{7}$

57. $5\sqrt{3} + 7\sqrt{12} = 3\sqrt{75}$

58. $5\sqrt{3} + 7\cdot2\sqrt{3} - 3\cdot5\sqrt{3} = 5\sqrt{3} + 14\sqrt{3} - 15\sqrt{3} = (5+14-15)\sqrt{3} = 4\sqrt{3}$

59. $13\sqrt{2} + 2\sqrt{18} - 5\sqrt{32}$

60. $\sqrt{8} - 3\sqrt{50} + 9\sqrt{32}$

61. $\sqrt{6} \cdot \sqrt{10} = \sqrt{2\cdot5\cdot\sqrt{2}} = \sqrt{4\cdot15} = 2\sqrt{15}$

62. $\sqrt{18} = 3\sqrt{2} \cdot \sqrt{3}$

63. $\sqrt{10} \cdot \sqrt{20} = \sqrt{200} = \sqrt{100\cdot2} = 10\sqrt{2}$

64. $\sqrt{11} \cdot \sqrt{33} = \sqrt{11\cdot11\cdot\sqrt{3}} = 11\sqrt{3}$

65. $\frac{\sqrt{8}}{\sqrt{4}} = \sqrt{2}$

66. $\frac{\sqrt{125}}{\sqrt{5}} = \frac{\sqrt{25}}{5} = 5$

67. $\frac{\sqrt{72}}{\sqrt{8}} = \frac{\sqrt{9}}{3} = 3$

68. $\frac{\sqrt{136}}{\sqrt{8}} = \frac{\sqrt{17}}{0}$
69. \( \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2} \)

70. \( \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3} \)

71. \( \frac{\sqrt{3} \cdot \sqrt{7}}{7} = \frac{\sqrt{21}}{7} \)

72. \( \frac{\sqrt{3}}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{30}}{\sqrt{100}} = \frac{\sqrt{30}}{10} \)

73. \( \frac{\sqrt{20}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{60}}{\sqrt{3}} = \frac{2\sqrt{15}}{3} \)

74. \( \frac{\sqrt{50}}{\sqrt{14}} = \frac{\sqrt{50}}{\sqrt{14}} = \frac{25}{7} \)

75. \( \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \)

76. \( \frac{\sqrt{15}}{\sqrt{3}} = \sqrt{5} \)

77. \( \frac{\sqrt{10}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{60}}{\sqrt{6}} = \frac{2\sqrt{15}}{6} = \frac{\sqrt{15}}{3} \)

78. \( \frac{8}{\sqrt{8}} = \frac{8\sqrt{2}}{\sqrt{16}} = \frac{8\sqrt{2}}{4} = 2\sqrt{2} \)

79. \( \sqrt{7} \) is between 2 and 3 since \( \sqrt{7} \) is between \( \sqrt{4} = 2 \) and \( \sqrt{9} = 3 \). \( \sqrt{7} \) is between 2.5 and 3 since 7 is closer to 9 than to 4. Using a calculator \( \sqrt{7} \approx 2.6 \).

80. \( \sqrt{37} \) is between 6 and 7 since \( \sqrt{37} \) is between \( \sqrt{36} = 6 \) and \( \sqrt{49} = 7 \). \( \sqrt{37} \) is between 6 and 6.5 since 37 is closer to 36 than to 49. Using a calculator \( \sqrt{37} \approx 6.1 \).

81. \( \sqrt{107} \) is between 10 and 11 since \( \sqrt{107} \) is between \( \sqrt{100} = 10 \) and \( \sqrt{121} = 11 \). \( \sqrt{107} \) is between 10 and 10.5 since 107 is closer to 100 than to 121. Using a calculator \( \sqrt{107} \approx 10.3 \).

82. \( \sqrt{135} \) is between 11 and 12 since \( \sqrt{135} \) is between \( \sqrt{121} = 11 \) and \( \sqrt{144} = 12 \). \( \sqrt{135} \) is between 11.5 and 12 since 135 is closer to 144 than to 121. Using a calculator \( \sqrt{135} \approx 11.6 \).

83. \( \sqrt{170} \) is between 13 and 14 since \( \sqrt{170} \) is between \( \sqrt{169} = 13 \) and \( \sqrt{196} = 14 \). \( \sqrt{170} \) is between 13 and 13.5 since 170 is closer to 169 than to 196. Using a calculator \( \sqrt{170} \approx 13.04 \).

84. \( \sqrt{200} \) is between 14 and 15 since \( \sqrt{200} \) is between \( \sqrt{196} = 14 \) and \( \sqrt{225} = 15 \). \( \sqrt{200} \) is between 14 and 14.5 since 200 is closer to 196 than to 225. Using a calculator \( \sqrt{200} \approx 14.1 \).

85. False, \( \sqrt{p} \) is an irrational number for any prime number \( p \).

86. False, The result may be a rational number or an irrational number.

87. True

88. True

89. False, The result may be a rational number or an irrational number.

90. False, The result may be a rational number or an irrational number.

91. \( \sqrt{2} + (-\sqrt{2}) = 0 \)

92. \( \sqrt{3} + 5\sqrt{3} = 6\sqrt{3} \)

93. \( \sqrt{2} \cdot \sqrt{3} = \sqrt{6} \)
94. \(\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3\)  
95. No. \(\sqrt{3} \neq 1.732\) since \(\sqrt{3}\) is an irrational number and 1.732 is a rational number.  

96. \(\sqrt{14} = \sqrt{7} \sqrt{2}\). \(\sqrt{7}\) is irrational and \(\sqrt{2}\) is irrational, therefore \(\sqrt{14}\) is irrational. Because 3.742 is rational, then \(\sqrt{14} \neq 3.742\).  

97. No. \(\frac{22}{7}\) are rational numbers, \(\pi\) is an irrational number.  

98. \(\sqrt{25} \neq 3 + 4\)  
\(\frac{5}{7} \neq 7\)  
\(T = 2\pi \sqrt{\frac{35}{980}} = 2\pi \frac{\sqrt{35}}{\sqrt{980}} = 2\pi \frac{\sqrt{5}\sqrt{7}}{\sqrt{5}\sqrt{196}}\)  
100. \(\sqrt{9} + 16 \neq \sqrt{9} + \sqrt{16}\)  
101. a) \(s = \frac{4}{0.04} = \frac{100}{1} = 10\) mph 
   b) \(s = \frac{16}{0.04} = \frac{400}{1} = 20\) mph 
   c) \(s = \frac{64}{0.04} = \frac{1600}{1} = 40\) mph 
   d) \(s = \frac{256}{0.04} = \frac{6400}{1} = 80\) mph  

102. a) \(t = \frac{\sqrt{100}}{4} = \frac{10}{4} = 2.5\) sec  
   b) \(t = \frac{\sqrt{400}}{4} = \frac{20}{4} = 5\) sec  
   c) \(t = \frac{\sqrt{900}}{4} = \frac{30}{4} = 7.5\) sec  
   d) \(t = \frac{\sqrt{1600}}{4} = \frac{40}{4} = 10\) sec  

103. a) The number is rational if the result on the calculator is a terminating or repeating decimal number. Otherwise, the number is irrational.  
   b) Using a calculator, \(\sqrt{0.04} = 0.2\) a terminating decimal and thus it is rational.  
   c) Using a calculator, \(\sqrt{0.07} = 0.264575131\ldots\), thus it is irrational.  

104. No. The sum of two irrational numbers may not be irrational. (i.e. \(-\sqrt{3} + \sqrt{3} = 0\)  

105. a) \((44 + \sqrt{4}) + \sqrt{4} = (44 + 2) + 2 = 22 + 2 = 11\)  
    b) \((44 + 4) + \sqrt{4} = 11 + 2 = 13\)  
    c) \(4 + 4 + 4 + \sqrt{4} = 12 + 2 = 14\)  
    d) \(\sqrt{4} (4 + 4) + \sqrt{4} = 2(8) + 2 = 16 + 2 = 18\)  

Exercise Set 5.5  
1. The set of real numbers is the union of the rational numbers and the irrational numbers.  
2. All real numbers = R  
3. If the given operation is preformed on any two elements of the set and the result is an element of the set, then the set is closed under the given operation.  
4. The order in which two numbers are multiplied does not make a difference in the result. Ex. \(2 \cdot 3 = 3 \cdot 2\)  
5. The order in which two numbers are added does not make a difference in the result. Ex. \(a + b = b + a\)
6. The associative property of addition states that when adding three real numbers, parentheses may be placed around any two adjacent numbers. \((a+b)+c = a+(b+c)\)

7. The associative property of multiplication states that when multiplying three real numbers, parentheses may be placed around any two adjacent numbers. Ex. \((2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)\).

8. The distributive property of multiplication over addition allows you to either add first and then multiply, or multiply first and then add. \(a(b+c) = ab + ac\)

9. Closed. The sum of two natural numbers is a natural number.

10. Not closed. (i.e. \(3 - 5 = -2\) is not a natural number).

11. Not closed. (i.e. \(3 + 5 = \frac{3}{5}\) is not a natural number).

12. Closed. The product of two natural numbers is a natural number.

13. Closed. The difference of two integers is an integer.

14. Closed. The sum of two integers is an integer.

15. Not closed. (i.e. \(2 + 5 = \frac{2}{5}\) = 0.4 is not an integer).

16. Closed. The product of two integers is an integer.


29. Commutative property. The order is changed from \((x) + (3 + 4) = (3 + 4) + x\).

30. \(4 + (5 + 6) = 4 + (6 + 5)\); Commutative because the only thing that has changed is the order of 5 and 6.

31. \((-4) \cdot (-5) = 20 = (-5) \cdot (-4)\)

32. \((-2) + (-3) = -5 = (-3) + (-2)\)

33. No. \(6 \cdot 3 = 2, \) but \(3 \cdot 6 = \frac{1}{2}\)

34. No. \(5 - 3 = 2, \) but \(3 - 5 = -2\)

35. \([(-3) \cdot (-5)] \cdot (-7) = (15) \cdot (-7) = -105\)

\((-3) \cdot \{[(-5)] \cdot (-7)] = (-3) \cdot (35) = -105\)

36. \([(-3) + (-5)] + (-7) = (-8) + (-7) = -15\)

\((-3) + [(-5)] + (-7)] = (-3) + (-12) = -15\)

37. No. \((8 + 4) \cdot 2 = 2 \cdot 2 = 1, \) but \(8 \cdot (4 + 2) = 8 \cdot 2 = 4\)

38. No. \((8 - 7) - 12 = 1 - 12 = -11, \) but \(8 - (-5) = 8 + 5 = 13\)

39. No. \((8 + 4) \cdot 2 = 2 \cdot 2 = 1, \) but \(8 \cdot (4 + 2) = 8 \cdot 2 = 4\)

40. No. \(2 + (3 \cdot 4) = 2 + 12 = 14, \) but \((2 + 3) \cdot (2 + 4) = 5 \cdot 6 = 30\)

41. Commutative property of addition

42. \((7 \cdot 4) \cdot 5 = (4 \cdot 5)\)

43. Associative property of multiplication

44. \(v + w = w + v\)

45. \((24 + 7) + 3 = 24 + (7 + 3)\)

46. \(4 \cdot (11 \cdot x) = (4 \cdot 11) \cdot x\)

47. Associative property of addition
47. $\sqrt{3} \cdot 7 = 7 \cdot \sqrt{3}$  
   Commutative property of multiplication

48. $\frac{3}{8} \left( \frac{1}{8} + \frac{3}{2} \right) = \left( \frac{3}{8} + \frac{1}{8} \right) + \frac{3}{2}$  
   Associative property of addition

49. $8(7 + \sqrt{2}) = 8 \cdot 7 + 8 \cdot \sqrt{2}$  
   Distributive property

50. $\sqrt{5} \cdot \frac{2}{3} = \frac{2}{3} \cdot \sqrt{5}$  
   Commutative property of multiplication

51. Commutative property of addition
52. Commutative property of addition
53. Distributive property
54. Commutative property of multiplication
55. Commutative property of addition
56. Commutative property of multiplication

57. $2(c + 7) = 2c + 14$  
   Associative property of addition

58. $-3(d - 1) = -3d + 3$  

59. $\frac{2}{3} (x - 6) = \frac{2}{3} x - \frac{12}{3} = \frac{2}{3} x - 4$  

60. $\frac{-5}{8} (k + 8) = \frac{-5}{8} k + \frac{-40}{8} = \frac{-5}{8} k = 5$

61. $6 \left( \frac{x + 2}{3} \right) = \frac{6x}{2} + \frac{12}{3} = 3x + 4$  

62. $24 \left( \frac{x - 1}{3} \right) = \frac{24x}{3} - \frac{24}{8} = 8x - 3$

63. $32 \left( \frac{1}{16} x - \frac{1}{32} \right) = \frac{32x}{16} - \frac{32}{32} = 2x - 1$  

64. $15 \left( \frac{2}{3} x - \frac{4}{5} \right) = \frac{30x}{3} - \frac{60}{5} = 10x - 12$

65. $3(5 - \sqrt{5}) = 15 - 3\sqrt{5}$  

66. $-7(2 + \sqrt{11}) = -14 - 2\sqrt{11}$

67. $\sqrt{2} (\sqrt{2} + \sqrt{3}) = \sqrt{4} + \sqrt{6} = 2 + \sqrt{6}$

68. $\sqrt{3} (\sqrt{15} + \sqrt{21}) = \sqrt{45} + \sqrt{63} = \sqrt{9 \sqrt{5}} + \sqrt{9 \sqrt{7}}$
   $= 3\sqrt{5} + 3\sqrt{7}$

69. a) Distributive property  
   b) Associative property of addition  
   c) Combine like terms

70. a) Distributive property  
   b) Associative property of addition;  
   c) Combine like terms

71. a) Distributive property  
   b) Associative property of addition;  
   c) Commutative property of addition  
   d) Associative property of addition  
   e) Combine like terms

72. a) Distributive property  
   b) Associative property of addition;  
   c) Commutative property of addition  
   d) Associative property of addition  
   e) Combine like terms

73. a) Distributive property  
   b) Commutative property of addition;  
   c) Associative property of addition  
   d) Combine like terms  
   e) Commutative property of addition

74. a) Distributive property  
   b) Commutative property of addition;  
   c) Associative property of addition  
   d) Combine like terms  
   e) Commutative property of addition

75. Yes. You can either lock your door first or put on your seat belt first.

76. Yes. Can be done independently; no order needed
77. No. The clothes must be washed first before being dryed.
78. No. The PC must be turned on first before you can type a term paper.
79. Yes. Can be done in either order; either fill the car with gas or wash the windshield.
80. No. The lamp must be turned on first before reading a book.
81. Yes. The order of events does not matter.
82. No. The book must be read first, then write a report, then make a presentation.
83. Yes. The order does not matter.
84. Yes. The order does not matter.
85. Yes. The final result will be the same regardless of the order of the events.
86. No. The egg cannot be poured before it is cracked.
87. Yes. The meatloaf will taste the same regardless of the order the items are mixed.
88. Yes. The order does not matter.
89. No. (Man eating) tiger is a tiger that eats men, and man (eating tiger) is a man that is eating a tiger.
b) No. (Horse riding) monkey is a monkey that rides a horse, and horse (riding monkey) is a horse that rides a monkey.
c) Answers will vary.

Exercise Set 5.6

1. 2 is the base and 3 is the exponent or power.
2. $b^n$ is $b$ multiplied by itself $n$ times. $b^n = \underbrace{b \cdot b \cdot \cdots \cdot b}_{n \text{ factors of } b}$

3. a) If $m$ and $n$ are natural numbers and $a$ is any real number, then $a^m \cdot a^n = a^{m+n}$
b) $2^3 \cdot 2^4 = 2^{3+4} = 2^7 = 128$

4. a) If $m$ and $n$ are natural numbers and $a$ is any real number except 0, then $\frac{a^n}{a^m} = a^{n-m}$.
b) $\frac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$

5. a) If $a$ is any real number except 0, then $a^0 = 1$.
b) $7^0 = 1$

6. a) If $n$ is a natural number and $a$ is any real number except 0, then $a^{-n} = \frac{1}{a^n}$.
b) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
7. a) If \( m \) and \( n \) are natural numbers and \( a \) is any real number, then \( (a^m)^n = a^{mn} \)

b) \( (3^2)^4 = 3^{2*4} = 3^8 = 6561 \)

8. Since 1 raised to any power equals 1, \( 1^{500} = 1 \).

9. a) Since 1 raised to any exponent equals +1, then \( -1^{500} = (-1)(1^{500}) = (-1)(1) = 1 \)

b) Since –1 raised to an even exponent equals 1, then number \( -1^{500} = \left((-1)^2\right)^{250} = (1)^{250} = 1 \)

c) In \( -1^{501} \) -1 is not raised to the \( 501^{th} \) power, but +1 is; so \( -1^{501} = (-1)(1^{501}) = (-1)(1) = -1 \)

d) Since –1 is raised to a negative exponent is -1, then \( (-1)^{-501} = -1 \)

10. a) Move the decimal point in the original number to the right or left until you obtain a number greater or equal to 1 and less than 10. Count the number of places the decimal was moved. If it was moved to the left the count is a positive number and if it was moved to the right the count is a negative number. Multiply the number obtained in the first step by 10 raised to the count number.

b) \( 0.000426 = 4.26 \times 10^{-4} \). note: the count number is \(-4\)

11. a) If the exponent is positive, move the decimal point in the number to the right the same number of places as the exponent adding zeros where necessary. If the exponent is negative, move the decimal point in the number to the left the same number of places as the exponent adding zeros where necessary.

b) \( 5.76 \times 10^{-4} = 0.000576 \)

12. a) The number is greater than or equal to 10.

b) The number is greater than or equal 1 but < 10.

c) The number is less than 1.

13. \( 5^2 = 5 \times 5 = 25 \)

14. \( 3^4 = 3 \times 3 \times 3 \times 3 = 81 \)

15. \( (-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16 \)

16. \( -2^4 = -2 \times 2 \times 2 \times 2 = -16 \)

17. \( -3^2 = -3 \times -3 = -9 \)

18. \( (-3)^2 = (-3) \times (-3) = 9 \)

19. \( \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \frac{4}{9} \)

20. \( \left(\frac{-7}{8}\right)^2 = \left(\frac{-7}{8}\right) \times \left(\frac{-7}{8}\right) = \frac{49}{64} \)

21. \( (-5)^2 = (-5) \times (-5) = 25 \)

22. \( -(5)^2 = -(5) \times (5) = -25 \)

23. \( 2^3 \times 3^2 = (2) \times (2) \times (2) \times (3) \times (3) = 72 \)

24. \( \frac{15^2}{3^2} = \frac{15 \times 15}{3 \times 3} = \frac{225}{9} = 25 \)
25. \[ \frac{5^7}{5^3} = 5^{7-3} = 5^2 = 5 \times 5 = 25 \]

26. \[ 3^1 \times 3^4 = 3^{1+4} = 3^7 = 2187 \]

27. \[ \frac{7^3}{7^1} = 7^{3-1} = 7^2 = \frac{1}{7^1} \times 7 = \frac{1}{49} \]

28. \[ 3^1 \times 7^0 = (3)(3)(3)(3)(1) = 81 \]

29. \[ (-3)^0 = 1 \]

30. \[ (-3)^4 = (-3)(-3)(-3)(-3) = 81 \]

31. \[ 3^4 = (3)(3)(3)(3) = 81 \]

32. \[ -3^4 = -(3)(3)(3)(3) = -81 \]

33. \[ 3^2 = \frac{1}{3^2} = \frac{1}{9} \]

34. \[ 3^{-3} = \frac{1}{3^3} = \frac{1}{27} \]

35. \[ (2^3)^4 = 2^{3\times4} = 2^{12} = 4096 \]

36. \[ (1^{12})^{13} = 1^{12 \times 13} = 1^{156} = 1 \]

37. \[ \frac{11^{25}}{11^{23}} = 11^{25-23} = 11^2 = 121 \]

38. \[ 5^2 \times 5 = 5^{2+1} = 5^3 = 125 \]

39. \[ (-4)^2 = (-4) \times (-4) = 16 \]

40. \[ 4^{-2} = \frac{1}{4^2} = \frac{1}{16} \]

41. \[ -4^2 = -(4) \times (4) = -16 \]

42. \[ (4^3)^2 = 4^{3\times2} = 4^6 = 4096 \]

43. \[ (2^3)^{-3} = 2^{3 \times (-3)} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64} \]

44. \[ 3^{-3} \times 3 = 3^{3+1} \times 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \]

45. \[ 231000 = 2.31 \times 10^5 \]

46. \[ 297000000 = 2.97 \times 10^8 \]

47. \[ 15 = 1.5 \times 10^1 \]

48. \[ 0.000034 = 3.4 \times 10^{-5} \]

49. \[ 0.56 = 5.6 \times 10^{-1} \]

50. \[ 0.00467 = 4.67 \times 10^{-3} \]

51. \[ 19000 = 1.9 \times 10^4 \]

52. \[ 1260000000 = 1.26 \times 10^9 \]

53. \[ 0.000186 = 1.86 \times 10^{-4} \]

54. \[ 0.00003 = 3.0 \times 10^{-4} \]

55. \[ 0.000000423 = 4.23 \times 10^{-6} \]

56. \[ 54000 = 5.4 \times 10^4 \]

57. \[ 711 = 7.11 \times 10^2 \]

58. \[ 0.02 = 2.0 \times 10^{-2} \]

59. \[ 0.153 = 1.53 \times 10^{-1} \]

60. \[ 416000 = 4.16 \times 10^5 \]

61. \[ 8.4 \times 10^4 = 84000 \]

62. \[ 2.71 \times 10^{-3} = 0.00271 \]

63. \[ 1.2 \times 10^{-2} = 0.012 \]

64. \[ 5.19 \times 10^5 = 519000 \]

65. \[ 2.13 \times 10^{-3} = 0.00213 \]

66. \[ 2.74 \times 10^{-7} = 0.000000274 \]

67. \[ 3.12 \times 10^{-1} = 0.312 \]

68. \[ 4.6 \times 10^4 = 46 \]

69. \[ 9.0 \times 10^6 = 9000000 \]

70. \[ 7.3 \times 10^4 = 73000 \]

71. \[ 2.31 \times 10^2 = 231 \]

72. \[ 1.04 \times 10^{-2} = 0.0104 \]

73. \[ 3.5 \times 10^4 = 35000 \]

74. \[ 2.17 \times 10^{-6} = 0.00000217 \]

75. \[ 1.0 \times 10^4 = 10000 \]

76. \[ 1.0 \times 10^{-3} = 0.001 \]
77. \[(2.0 \times 10^3)(4.0 \times 10^2) = 8.0 \times 10^5 = 800000\]
79. \[(5.1 \times 10^3)(3.0 \times 10^{-3}) = 15.3 \times 10^{-3} = 0.0153\]
81. \[6.4 \times 10^5 \frac{2.0 \times 10^3}{2.0 \times 10^3} = 3.2 \times 10^2 = 320\]
83. \[8.4 \times 10^{-6} \frac{4.0 \times 10^{-3}}{4.0 \times 10^{-3}} = 2.1 \times 10^{-3} = 0.0021\]
85. \[4.0 \times 10^5 \frac{2.0 \times 10^3}{2.0 \times 10^3} = 2.0 \times 10^3 = 20\]
87. \[(3.0 \times 10^{-3})(2000000) = (3.0 \times 10^3)(2.0 \times 10^6) = 6.0 \times 10^{11}\]
89. \[(3.0 \times 10^{-3})(1.5 \times 10^{-4}) = 4.5 \times 10^{-7}\]
91. \[\frac{1.4 \times 10^6}{7.0 \times 10^3} = 0.2 \times 10^4 = 2.0 \times 10^3\]
93. \[\frac{4.0 \times 10^{-5}}{2.0 \times 10^{-7}} = 2.0 \times 10^2\]
95. \[\frac{1.5 \times 10^5}{5.0 \times 10^3} = 0.3 \times 10^3 = 3.0 \times 10^3\]
97. \[8.3 \times 10^{-4}, 3.2 \times 10^{-1}, 4.6, 5.8 \times 10^5\]
99. \[8.3 \times 10^{-5}, 0.00079; 4.1 \times 10^3; 40,000;\] Note: \[0.00079 = 7.9 \times 10^{-4}, 40,000 = 4 \times 10^4\]
101. \[1,962,000; 4.79 \times 10^6; 3.14 \times 10^7; 267,000,000\]
103. \[\frac{10.1432 \times 10^{12}}{285.0 \times 10^6} = 0.3559017548 \times 10^6\]
105. \[\frac{4.1468 \times 10^{12}}{127.0 \times 10^6} = 0.0326514685 \times 10^6\]
107. \[\frac{7.69 \times 10^{33}}{36.6 \times 10^{32}} = 0.2101092896 \times 10^{21}\]
109. \[6.251 \times 10^9 - 1.283 \times 10^9 = 4.968 \times 10^9\]
111. \[t = \frac{d}{r} = \frac{4.5 \times 10^8}{2.5 \times 10^7} = 1.8 \times 10^4\]
113. \[t = \frac{d}{r} = \frac{2390000 \text{ mi}}{20000 \text{ mph}} = 11.95\]
115. \[6.8 \times 10^{-1} \times 3.0 \times 10^0 = 2.0 \times 10^0\]
117. \[6.7 \times 10^{-1} \times 4.0 \times 10^3 = 2.7 \times 10^2\]
119. \[8.5 \times 10^{20} \times 3.6 \times 10^{-20} = 3.0 \times 10^0\]
121. \[6.3 \times 10^9 \times 3.14 \times 10^0 = 2.0 \times 10^{10}\]
123. \[6.2 \times 10^4 \times 3.6 \times 10^{-1} = 2.3 \times 10^3\]
107. \((500,000)(40,000,000,000) = (5 \times 10^5)(4 \times 10^{10}) = 20 \times 10^{15} = 2 \times 10^{16}\)
   a) 20,000,000,000,000,000 drops    b) 2.0 \times 10^{16} drops

108. \((50)(5,800,000) = (5 \times 10^1)(5.8 \times 10^6) = 29 \times 10^7 = 2.9 \times 10^8\)
   a) 290,000,000 cells    b) 2.9 \times 10^8 cells

109. \(\frac{4.5 \times 10^9}{2.5 \times 10^5} = 1.8 \times 10^4\)
   a) 18,000 times    b) 1.8 \times 10^4 times

110. a) \((100,000 \text{ cu.ft./sec}) (60 \text{ sec/min}) (60 \text{ min/hr}) (24 \text{ hr}) = 8,640,000,000 \text{ ft}^3\)    b) 8.64 \times 10^9 \text{ cu ft}

111. \(\frac{4.65 \times 10^{12}}{257.0 \times 10^6} = 0.0180933852 \times 10^6\)
   a) $32,651.97 \approx 18,093.00 = $3,434.78

112. a) 18 billion = 18,000,000,000 = 1.8 \times 10^{10} \text{ diapers}
   b) \((14) (2.38) (10^5) = 33.32 \times 10^5 = 3.332 \times 10^6 \text{ or 3,332,000 miles}

113. a) \((0.60)(1,200,000,000) = $720,000,000\)
   b) \((0.25)(1,200,000,000) = $300,000,000\)
   c) \((0.10)(1,200,000,000) = $120,000,000\)
   d) \((0.05)(1,200,000,000) = $60,000,000\)

114. a) \((0.40)(3,400,000,000) = $1,360,000,000\)
   b) \((0.40)(3,400,000,000) = $1,360,000,000\)
   c) \((0.10)(3,400,000,000) = $340,000,000\)
   d) \((0.10)(3,400,000,000) = $340,000,000\)

115. 1,000 times, since 1 meter = 10^3 millimeters = 1,000 millimeters

116. Since 1 gram = \(10^3\) milligrams and 1 gram = \(10^{-3}\) kilograms, \(10^{-3}\) kilograms = \(10^3\) milligrams
    \(\frac{10^{-3} \text{ Kilograms}}{10^3} = \frac{10^3 \text{ milligrams}}{10^3}\), Thus, 1 kilogram = \(10^6\) milligrams

117. \(\frac{2 \times 10^{30}}{6 \times 10^{24}} = 0.3 \times 10^6 = 333,333 \text{ times}\)

118. a) \((2) (6 \text{ billion}) = 12 \text{ billion} = 12,000,000,000 \text{ people}\)
   b) \(\frac{6,000,000,000}{(35)(365)} = \frac{6,000,000,000}{12775} = 469,667 \text{ people per day}\)

119. \(\frac{897,000,000,000,000}{3,900,000,000,000} = \frac{8.97 \times 10^{17}}{3.9 \times 10^{12}} = 2.3 \times 10^5 = 230,000 \text{ seconds or about 2.66 days}\)
120. a) \(1,000,000 = 1.0 \times 10^6\); \(1,000,000,000 = 1.0 \times 10^9\); \(1,000,000,000,000 = 1.0 \times 10^{12}\)

b) \(\frac{1.0 \times 10^6}{1.0 \times 10^3} = 1.0 \times 10^3\) days or 1,000 days = 2.74 years

c) \(\frac{1.0 \times 10^9}{1.0 \times 10^3} = 1.0 \times 10^6\) days or 1,000,000 days = 2,739.73 years

d) \(\frac{1.0 \times 10^{12}}{1.0 \times 10^3} = 1.0 \times 10^9\) days or 1,000,000,000 days = 2,739,726.03 years

e) \(\frac{1 \text{ billion}}{1 \text{ million}} = \frac{1.0 \times 10^9}{1.0 \times 10^6} = 1.0 \times 10^3 = 1,000\) times greater

121. a) \((1.86 \times 10^5 \text{ mi/sec}) (60 \text{ sec/min}) (60 \text{ min/hr}) (24 \text{ hr/day}) (365 \text{ days/yr}) (1 \text{ yr})\)

\[= (1.86 \times 10^5)(6 \times 10^1)(6 \times 10^1)(2.4 \times 10^1)(3.65 \times 10^2) = 586.5696 \times 10^{10} = 5.865696 \times 10^{12} \text{ miles}\]

b) \(t = \frac{d}{r} = \frac{9.3 \times 10^7}{1.86 \times 10^5} = 5.0 \times 10^2 = 500\) seconds or 8 min. 20 sec.

122. a) \(E(0) = 2^{10} \times 2^0 = 2^{10} \times 1 = 1024\) bacteria

b) \(E(1/2) = 2^{10} \times 2^{1/2} = 2^{10.5} = 1448.2\) bacteria

**Exercise Set 5.7**

1. A sequence is a list of numbers that are related to each other by a given rule. One example is 2, 4, 6, 8,....

2. The terms of the sequence.

3. a) An arithmetic sequence is a sequence in which each term differs from the preceding term by a constant amount. One example is 1, 4, 7, 10,....

b) A geometric sequence is one in which the ratio of any term to the term that directly precedes it is a constant.

One example is 1, 3, 9, 27,....

4. a) \(d = +3\), b) \(r = \frac{2}{1}\)

5. a) \(a_n = n^{\text{th}}\) term of the sequence b) \(a_1 = 1^{\text{st}}\) term of a sequence c) \(d = \) common difference in a sequence d) \(s_n = \) the sum of the 1\(^{st}\) n terms of the arithmetic sequence

6. a) \(a_n = n^{\text{th}}\) term of the sequence b) \(a_1 = 1^{\text{st}}\) term of a sequence c) \(r = \) common ratio between consecutive terms d) \(s_n = \) the sum of the 1\(^{st}\) n terms of the arithmetic sequence

7. \(a_1 = 3, d = 2\) \(3, 5, 7, 9, 11\)

8. \(a_1 = 1, d = 3\) \(1, 4, 7, 10, 13\)

9. \(a_1 = -5, d = -3\) \(-5, -2, 1, 4, 7\)

10. \(a_1 = -11, d = -6\) \(-11, -6, -1, 4, 9\)

11. \(5, 3, 1, -1, -3\)

12. \(-3, -7, -11, -15, -19\)

13. \(1/2, 1, 3/2, 2, 5/2\)

14. \(5/2, 1, -1/2, -2, -7/2\)

15. \(a_6, a_7 = 2, d = 3\) \(2, 5, 8, 11, 17\) \(a_6 = 17\)

16. \(a_9, a_1 = 3, d = -2\) \(-3, 1, -1, -3, -5, -7, -9, -11, -13\) \(a_9 = -13\)
17. \(a_{10}, a_1 = -5, d = 2, a_{10} = 13\)

18. \(a_{12} = 7 + (12 - 1)(-3) = 7 + (11)(-3) = 7 - 33 = -26\)

19. \(a_{12} = 7 + (12 - 1)(-3) = 7 + 11(-3) = 7 - 33 = -26\)

20. \(-\frac{1}{2} + (14)(-2) = -\frac{1}{2} - 28 = -\frac{1}{2} - 56 = -\frac{57}{2}\)

21. \(a_{11} = 4 + (10) \left(\frac{1}{2}\right) = 4 + 5 = 9\)

22. \(a_{18} = 4 + (14) \left(\frac{1}{3}\right) = \frac{4}{3} + \frac{14}{3} = \frac{18}{3} = 6\)

23. \(a_n = n, a_1 = 1 + (n - 1)1 = 1 + n - 1 = n\)

24. \(a_n = 2n - 1, a_n = 1 + (n - 1)2 = 1 + 2n - 2 = 2n - 1\)

25. \(a_n = 2n, a_n = 2 + (n - 1)2 = 2 + 2n - 2 = 2n\)

26. \(3, 1, -1, -3, a_n = 3 + (n - 1)(-2) = 3 - 2n + 2\)

27. \(a_n = -\frac{5}{3} + (n - 1)\left(\frac{1}{3}\right) = -\frac{5}{3} + \frac{1}{3} - \frac{1}{3} = -\frac{n - 2}{3}\)

28. \(a_n = -15 + (n - 1)(5) = 5n - 20\)

29. \(a_n = -3 + (n - 1)\left(\frac{3}{2}\right) = -3 + \frac{3}{2} - \frac{3}{2} = \frac{3}{2} - 9\)

30. \(a_n = -5 + (n - 1)(3) = 3n - 8\)

31. \(s_n = \frac{n(a_1 + a_n)}{2} = \frac{50(1 + 50)}{2} = \frac{50(51)}{2} = (25)(51) = 1275\)

32. \(s_n = \frac{50(2 + 100)}{2} = \frac{50(102)}{2} = (25)(102) = 2550\)

33. \(s_n = \frac{50(1 + 99)}{2} = \frac{50(100)}{2} = (25)(100) = 2500\)

34. \(s_n = \frac{9(-4 + (-28))}{2} = \frac{9(-32)}{2} = -144\)

35. \(s_n = \frac{8(11 + (-24))}{2} = \frac{8(-13)}{2} = -52\)

36. \(s_n = \frac{18\left(-9 + \left(-\frac{1}{2}\right)\right)}{2} = \frac{18\left(-\frac{19}{2}\right)}{2} = \frac{-171}{2} = -85.5\)

37. \(s_n = \frac{8\left(\frac{1}{2} + \frac{29}{2}\right)}{2} = \frac{8\left(\frac{30}{2}\right)}{2} = \frac{8\cdot 15}{2} = 60\)

38. \(s_n = \frac{18\left(\frac{3}{5} + 4\right)}{2} = \frac{18\left(\frac{23}{5}\right)}{2} = \frac{207}{5} = 41.4\)

39. \(a_1 = 3, r = 2, a_n = a_1r^{n-1} = (3)(2)^{n-1}\)

40. \(a_1 = 6, r = 3, a_n = 6(3)^{n-1} = 6(81) = 486\)

41. \(a_1 = 2, r = -2, a_n = 2(-2)^{n-1} = 2(16) = 32\)

42. \(a_1 = 4, r = 2, a_n = 1\)

43. \(-3, 3, -3, 3, -3\)

44. \(-6, 12, -24, 48, -96\)

45. \(-16, 8, -4, 2, -1\)

46. \(5, 3, 9/5, 27/25, 81/125\)

47. \(a_6 = 3(4)^3 = (3)(1024) = 3072\)

48. \(a_5 = 2(2)^3 = (2)(16) = 32\)
49. \( a_3 = 3 \left( \frac{1}{2} \right)^3 = 3 \left( \frac{1}{4} \right) = \frac{3}{4} \)

50. \( a_7 = -3(-3)^6 = -3(729) = -2187 \)

51. \( a_5 = a \left( \frac{1}{4} \right)^4 = 3 \left( \frac{1}{2} \right)^4 = \frac{3}{16} \)

52. \( a_{25} \quad a_1 = 2, \quad r = 2, \quad a_{25} = (2)^{25} = 16,777,216 \)

53. \( a = \frac{4}{11} \)

54. \( a_{18} \quad a_1 = -5, \quad r = -2, \quad a_{18} = (5)(-2)^{18} = 655,360 \)

55. \( 1, 2, 4, 8 \quad a_n = 2^n \)

56. \( 3, 6, 12, 24 \quad a_n = 3n \)

57. \( 3, -3, 3, -3 \quad a_n = (-1)^{n+1} \)

58. \( a_n = a_n = (1/2)^{n-1} \)

59. \( a_n = a_n = (1/4)^{n-1} \)

60. \( a_n = a_n = (1/2)^{n-1} \)

61. \( a_n = a_n = (9) \left( \frac{1}{3} \right)^{n-1} \)

62. \( a_n = a_n = (4) \left( \frac{2}{3} \right)^{n-1} \)

63. \( S_4 = a_1 \frac{1}{1-r} \left( 1 - r^4 \right) = \frac{3(1-2^4)}{1-2} = \frac{3(-15)}{-1} = 45 \)

64. \( S_8 = a_1 \frac{1}{1-r} \left( 1 - r^5 \right) = \frac{2(1-3^5)}{1-3} = \frac{2(-242)}{-2} = 242 \)

65. \( S_7 = a_1 \frac{1}{1-r} \left( 1 - r^7 \right) = \frac{5(1-4^7)}{1-4} = \frac{5(-6383)}{-3} = 27,305 \)

66. \( S_9 = a_1 \frac{1}{1-r} \left( 1 - r^9 \right) = \frac{3(1-5^9)}{1-5} = \frac{3(-953124)}{-4} = -1,464,843 \)

67. \( n = 15, \ a_1 = -1, \ r = 2 \)

68. \( n = 10, \ a_1 = 512, \ r = \frac{1}{2} \)

69. \( n = 15, \ a_1 = -1, \ r = -2 \)

70. \( n = 10, \ a_1 = 512, \ r = \frac{1}{2} \)

71. \( S_{15} = \frac{(100)(1+100)}{2} = \frac{(100)(101)}{2} = 50(101) = 5050 \)

72. \( S_{10} = \frac{(100)(2+200)}{2} = \frac{(100)(202)}{2} = 50(202) = 10100 \)

73. \( S_{10} = \frac{(100)(1+199)}{2} = \frac{(100)(200)}{2} = 50(200) = 10000 \)

74. \( S_{50} = \frac{(50)(3+150)}{2} = \frac{(50)(153)}{2} = 25(153) = 3825 \)
75. a) Using the formula \( a_n = a_1 + (n - 1)d \), we get
\[
a_8 = 20,200 + (8 - 1)(1200) = 28,600
\]
b) \[
\frac{20000 + 28600}{2} = \frac{48800}{2} = 19,400
\]
76. a) \( a_{12} = 96 + (11)(-3) = 96 - 33 = 63 \) in.
b) \[
\frac{[12(96+63)]}{2} = \frac{(12)(159)}{2} = (6)(159) = 954 \text{ in.}
\]
77. \( a_{11} = 72 + (10)(-6) = 72 - 60 = 12 \) in.
78. \( s_{12} = \frac{12(1+12)}{2} = \frac{12(13)}{2} = \frac{156}{2} = 78 \) times
80. \( a_n = a_1 r^n \quad a_{15} = (8000)(1.08)^{15} = 15992 \text{ students} \)
81. \( a_6 = 200(0.8)^6 = 200(0.262144) = 52.4288 \text{ g} \)
82. \( a_{15} = 30(0.8)^{14} = 12.288 \text{ ft.} \)
83. \( a_{15} = 20,000(1.06)^{14} = 45,218 \)
84. \( s_5 = \frac{30(0.8)^4}{2} = 12.888 \text{ ft.} \)
85. This is a geometric sequence where \( a_1 = 2000 \) and \( r = 3 \). In ten years the stock will triple its value 5 times.
\[
a_6 = a_1 r^{b-1} = 2000(3)^5 = 486,000
\]
86. The sequence of bets during a losing streak is geometric.
a) \( a_6 = a_1 r^{b-1} = 1(2)^{6-1} = 1(32) = 32 \)
s\( s_5 = \frac{a_1(1-r^n)}{1-r} = \frac{1(1-2^5)}{1-2} = \frac{-31}{-1} = 31 \)
b) \( a_6 = a_1 r^{b-1} = 10(2)^{6-1} = 10(32) = 320 \)
s\( s_5 = \frac{a_1(1-r^n)}{1-r} = \frac{10(1-2^5)}{1-2} = \frac{10(-31)}{-1} = 310 \)
c) \( a_{11} = a_1 r^{b-1} = 1(2)^{11-1} = 1(1024) = 1,024 \)
s\( s_{10} = \frac{a_1(1-r^n)}{1-r} = \frac{1(1-2^{10})}{1-2} = \frac{1(-1023)}{-1} = 1,023 \)
d) \( a_{11} = a_1 r^{b-1} = 10(2)^{11-1} = 10(1024) = 10,240 \)
s\( s_{10} = \frac{a_1(1-r^n)}{1-r} = \frac{10(1-2^{10})}{1-2} = \frac{10(-1023)}{-1} = 10,230 \)
e) If you lose too many times in a row, then you will run out of money.
87. \[
\frac{82[1-(1/2)^6]}{1-(1/2)} = \frac{82[1-(1/64)]}{1/2} = \frac{82 \cdot 63 \cdot 2}{1} = 161,437.5
\]
88. The arithmetic sequence \( 180^0, 360^0, 540^0, 720^0, \ldots \) has a common difference of 180.
Thus, \( a_n = 180(n - 2) = 180n - 360, \ n \geq 3 \)
89. 12, 18, 24, \ldots, 1608 is an arithmetic sequence with \( a_1 = 12 \) and \( d = 6 \). Using the expression for the \( n^{th} \) term of an arithmetic sequence \( a_n = a_1 + (n - 1)d \) or \( 1608 = 12 + (n - 1)6 \) and dividing both sides by 6 gives \( 268 = 2 + n - 1 \) or \( n = 267 \)
90. Since \( a_5 = a_1 r^4 \) and \( a_2 = a_1 r \), \( a_2/a_2 = r^2 \). Thus \( r^2 = 648/24 = 27 \) or \( r = 3 \).
Then \( 24 = a_2 = a_1 r = a_1(3) \) or \( a_1 = 24/3 = 8 \).
91. The total distance is 30 plus twice the sum of the terms of the geometric sequence having $a_1 = (30)(0.8) = 24$ and $r = 0.8$. Thus $s_5 = \frac{24[1-(0.8)^5]}{(1-0.8)} = \frac{24[1-0.32768]}{0.2} = \frac{24(0.67232)}{0.2} = 80.6784$.

So the total distance is $30 + 2(80.6784) = 191.3568$ ft.

92. The sequence of bets during a losing streak is geometric.
   
   a) $a_6 = a_1r^{n-1} = 1(2)^{6-1} = 1(32) = $32
   
   b) $a_6 = a_1r^{n-1} = 10(2)^{6-1} = 10(32) = $320
   
   c) $a_{11} = a_1r^{n-1} = 1(2)^{11-1} = 1(1024) = $1,024
   
   d) $a_{11} = a_1r^{n-1} = 10(2)^{11-1} = 10(1024) = $10,240
   
   e) If you lose too many times in a row, then you will run out of money.

Exercise Set 5.8

1. Begin with the numbers 1, 1, then add 1 and 1 to get 2 and continue to add the previous two numbers in the sequence to get the next number in the sequence.

2. a) 1,2,3,5,8,13,21,34,55,89  
   b) $\frac{55}{34} = 1.61764 \rightarrow 1.619$  
   c) $\frac{89}{55} = 1.61818 \rightarrow 1.618$

   d) $\frac{8}{5} = 1.6 \rightarrow 1.600$  
   e) $\frac{5}{3} = 1.6 \rightarrow 1.667$  
   f) $\frac{21}{13} = 1.61538 \rightarrow 1.615$

3. a) Golden number = $\frac{\sqrt{5}+1}{2}$
   
   b) 1.618 = golden ratio  
   
   When a line segment AB is divided at a point C, such that the ratio of the whole, AB, to the larger part, AC, is equal to the ratio of the larger part, AC, to the smaller part, CB, then each of the ratios $\frac{AB}{AC}$ and $\frac{AC}{CB}$ is known as the golden ratio.
   
   c) The golden proportion is: $\frac{AB}{AC} = \frac{AC}{CB}$

   d) The golden rectangle: $L = a + b = \frac{a}{b} = \frac{\sqrt{5}+1}{2} =$ golden number

4. All are essentially the same number when rounded.

6. a) Petals on daisies  
   b) Parthenon in Athens

5. a) Flowering head of a sunflower  
   b) Great Pyramid

   7. a) $\frac{\sqrt{5}+1}{2} = 1.618033989$  
   b) $\frac{\sqrt{5}-1}{2} = 0.6180339887$

   c) Differ by 1

8. $\frac{1}{89} = 0.0112359551$, part of Fibonacci sequence
9. \( \frac{1}{1} = 1, \frac{2}{1} = 2, \frac{3}{2} = 1.5, \frac{5}{3} = 1.6, \frac{8}{5} = 1.6, \frac{13}{8} = 1.625, \frac{21}{13} = 1.6154, \frac{34}{21} = 1.619, \frac{55}{34} = 1.6176 \)
\( \frac{89}{55} = 1.61818 \). The consecutive ratios alternate increasing then decreasing about the golden ratio.

10. The ratio of the second to the first and the fourth to the third estimates the golden ratio.

11. 

<table>
<thead>
<tr>
<th>Fib. No.</th>
<th>prime factors</th>
<th>Fib. No.</th>
<th>prime factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-------</td>
<td>34</td>
<td>2 \cdot 17</td>
</tr>
<tr>
<td>1</td>
<td>-------</td>
<td>55</td>
<td>5 \cdot 11</td>
</tr>
<tr>
<td>2</td>
<td>prime</td>
<td>89</td>
<td>prime</td>
</tr>
<tr>
<td>3</td>
<td>prime</td>
<td>144</td>
<td>2^4 \cdot 3^2</td>
</tr>
<tr>
<td>5</td>
<td>prime</td>
<td>233</td>
<td>prime</td>
</tr>
<tr>
<td>8</td>
<td>2^3</td>
<td>377</td>
<td>13 \cdot 29</td>
</tr>
<tr>
<td>13</td>
<td>prime</td>
<td>610</td>
<td>2 \cdot 5 \cdot 61</td>
</tr>
</tbody>
</table>

12. If the first ten are selected:
\[
\frac{1+1+2+3+5+8+13+21+34+55}{11} = \frac{143}{11} = 13
\]

13. If 5 is selected the result is \( 2(5) - 8 = 10 - 8 = 2 \) which is the second number preceding 5.

14. If 2, 3, 5, and 8 are selected the result is \( 5^2 - 3^2 = 2 \cdot 8 \rightarrow 25 - 9 = 16 \rightarrow 16 = 16 \)

15. Answers will vary. 16. \( \frac{6}{4} = 1.5 \) which is a little < 1.6. 17. Answers will vary.
18. Answers will vary. 19. Answers will vary. 20. Answers will vary.
21. Answers will vary.

23. Fibonacci type; \( 11 + 18 = 29 \quad 18 + 29 = 47 \)
24. Not Fibonacci. Each term is not the sum of the two preceding terms.
25. Not Fibonacci. Each term is not the sum of the two preceding terms.
26. Fibonacci type; \( 1 + 2 = 3, \quad 2 + 3 = 5 \quad \) Each term is the sum of the two preceding terms.

27. Fibonacci type; \( 40 + 65 = 105; \quad 65 + 105 = 170 \)
28. Fibonacci type; \( \frac{1}{4} + 2 = 3\frac{1}{4}; \quad 2 + 3\frac{1}{4} = 5\frac{1}{4} \)

29. Fibonacci type; \(-1 + 0 = -1; \quad 0 + (-1) = -1 \)
30. Fibonacci type; \( 7 + 13 = 20; \quad 13 + 20 = 33 \)

31. a) If 6 and 10 are selected the sequence is \( 6, 10, 16, 26, 42, 68, 110, \ldots \)
\( \quad \) b) \( \frac{10}{6} = 1.666, \frac{16}{10} = 1.600, \frac{26}{16} = 1.625, \frac{42}{26} = 1.615, \frac{68}{42} = 1.619, \frac{110}{68} = 1.618, \ldots \)

32. a) If 5 and 7 are selected the sequence is \( 5, 7, 12, 19, 31, 50, 81, \ldots \)
\( \quad \) b) \( \frac{7}{5} = 1.4, \frac{12}{7} = 1.714, \frac{19}{12} = 1.583, \frac{31}{19} = 1.623, \frac{50}{31} = 1.613, \frac{81}{50} = 1.62, \ldots \)

33. a) If 5, 8, and 13 are selected the result is \( 8^2 - (5)(13) = 64 - 65 = -1 \).
\( \quad \) b) If 21, 34, and 55 are selected the result is \( 34^2 - (21)(55) = 1156 - 1155 = 1 \).
\( \quad \) c) The square of the middle term of three consecutive terms in a Fibonacci sequence differs from the product of the 1st and 2nd term by 1.
34. The sum of the numbers along the diagonals parallel to the one shown is a Fibonacci number.

35. a) Lucas sequence: 1, 3, 4, 7, 11, 18, 29, 47, …  
   b) \(8 + 21 = 29; \ 13 + 34 = 47\)  
   c) The first column is a Fibonacci-type sequence.

36. \(-10, x, -10 + x, -10 + 2x, -20 + 3x, -30 + 5x, -50 + 8x, -80 + 13x, -130 + 21x, -210 + 34x\)
   a) \(-10, 4, -6, -2, -8, -10, -18, -28, -46, -74\)
   b) \(-10, 5, -5, 0, -5, -5, -10, -15, -25, -40\)
   c) \(-10, 6, -4, 2, -2, 0, -2, -4, -6\)
   d) \(-10, 7, -3, 4, 1, 5, 6, 11, 17, 28\)
   e) \(-10, 8, -2, 6, 4, 10, 14, 28, 38, 62\)
   f) Yes, because each multiple causes the \(x\) term to be greater than the number term.

37. \(\frac{a+b}{ab} = \frac{a}{b} \) Let \(x = \frac{a}{b} \ \frac{b}{a} = \frac{1}{x} \ \frac{1+b}{a} = \frac{a}{b} \ \frac{1}{x} = \frac{x}{1} \) multiply by \(x\)
   \(x + 1 = x^2 \ \ x^2 - x - 1 = 0 \  \ a = 1, \ b = -1, \ c = -1\)

Solve for \(x\) using the quadratic formula, \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}\)

38. \(\frac{5-x}{5} = \frac{5}{x} \) \(x (5-x) = 25 \ 5x - x^2 = 25 \  x^2 - 5x + 25 = 0 \  \ a = 1, \ b = -5, \ c = 25\)

Solve for \(x\) using the quadratic formula, \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{25 - 4(1)(25)}}{2(1)} = \frac{5 \pm \sqrt{-75}}{2} = \frac{5 \pm 5\sqrt{3i}}{2}\)

39. Answers will vary. \{5, 12, 13\} \{16, 30, 34\} \{105, 208, 233\} \{272, 546, 610\}

40. a) 3 reflections, 5 paths  
   b) 4 reflections, 8 paths  
   c) 5 reflections, 13 paths

**Review Exercises**

1. Use the divisibility rules in section 5.1. 
   670,920 is divisible by 2, 3, 4, 5, 6, and 9.

2. Use the divisibility rules in section 5.1. 
   400,644 is divisible by 2, 3, 4, 6, and 9

3. \(\begin{array}{c}
2 & 252 \\
2 & 126 \\
3 & 63 \\
3 & 21 \\
7 & \\
\end{array}\)  
   \(252 = 2^2 \cdot 3^3 \cdot 7\)

4. \(\begin{array}{c}
5 & 385 \\
7 & 77 \\
11 & \\
\end{array}\)

5. \(\begin{array}{c}
2 & 840 \\
2 & 420 \\
2 & 210 \\
5 & 105 \\
3 & 21 \\
7 & \\
\end{array}\)

\(840 = 2^3 \cdot 3 \cdot 5 \cdot 7\)
6. \[
\begin{array}{c} \\
882 \\
441 \\
147 \\
49 \\
7 \\
\hline
2 \\
3 \\
7 \\
\end{array}
\]
\[
882 = 2 \cdot 3^2 \cdot 7^2
\]

7. \[
\begin{array}{c} \\
1452 \\
726 \\
363 \\
121 \\
11 \\
\hline
2 \\
2 \\
3 \\
11 \\
\end{array}
\]
\[
1452 = 2^2 \cdot 3^2 \cdot 11^2
\]

8. \[
15 = 3 \cdot 5, \quad 60 = 2^2 \cdot 3 \cdot 5
\]
\[
gcd = 15, \quad lcm = 60
\]

9. \[
63 = 3 \cdot 3 \cdot 5, \quad 108 = 3 \cdot 4 \cdot 9
\]
\[
gcd = 9, \quad lcm = 756
\]

10. \[
45 = 3^2 \cdot 5, \quad 250 = 2 \cdot 5^3; \quad gcd = 5, \quad lcm = 2 \cdot 3 \cdot 5^3 = 2250
\]

11. \[
840 = 2^3 \cdot 3 \cdot 5 \cdot 7, \quad 320 = 2^6 \cdot 5; \quad gcd = 2^3 \cdot 5 = 40; \quad lcm = 2^6 \cdot 3 \cdot 5 \cdot 7 = 6720
\]

12. \[
60 = 2^2 \cdot 3 \cdot 5, \quad 40 = 2^3 \cdot 5, \quad 96 = 2^5 \cdot 3; \quad gcd = 2^2 = 4; \quad lcm = 2^5 \cdot 3 \cdot 5 = 480
\]

13. \[
36 = 2^2 \cdot 3^2, \quad 108 = 2^2 \cdot 3^3, \quad 144 = 2^4 \cdot 3^2; \quad gcd = 2^2 \cdot 3^2 = 36; \quad lcm = 2^4 \cdot 3^3 = 432
\]

14. \[
15 = 3 \cdot 5, \quad 9 = 3^2; \quad lcm = 3^2 \cdot 5 = 45. \quad \text{In 45 days the train will stop in both cities.}
\]

15. \[
-2 + 5 = 3
\]

16. \[
4 + (-7) = -3
\]

17. \[
4 - 8 = 4 + (-8) = -4
\]

18. \[
(-2) + (-4) = -6
\]

19. \[
-5 - 4 = -5 + (-4) = -9
\]

20. \[
-3 - (-6) = -3 + 6 = 3
\]

21. \[
(-3 + 7) - 4 = 4 + (-4) = 0
\]

22. \[
-1 + (9 - 4) = -1 + 5 = 4
\]

23. \[
(-3)(-11) = 33
\]

24. \[
(-4)(9) = -36
\]

25. \[
14(-3) = -56
\]

26. \[
-35/7 = 5
\]

27. \[
12/6 = 2
\]

28. \[
[8 + (-4)](-3) = (-2)(-3) = 6
\]

29. \[
[(−4)(−3)] + 2 = 12 + 2 = 6
\]

30. \[
[−30 + (10)] \div (−1) = −3 + (−1) = 3
\]

31. \[
3/10 = 0.3
\]

32. \[
3/5 = 0.6
\]

33. \[
15/40 = 3/8 = 0.375
\]

34. \[
13/4 = 3.25
\]

35. \[
3(7) = 0.428571
\]

36. \[
7/12 = 0.583
\]

37. \[
3/8 = 0.375
\]

38. \[
7/8 = 0.875
\]

39. \[
5/7 = 0.714285
\]

40. \[
0.225 = \frac{225}{1000} = \frac{45}{200} = \frac{9}{40}
\]

41. \[
4.5 = \frac{9}{2}
\]

42. \[
0.6666 \quad 10n = 6.6666….
\]

\[
10n = 6.6
\]

\[
9 \quad 9
\]

\[
9n = 6.0
\]

\[
n = \frac{2}{3}
\]

43. \[
2.373737 \quad 100n = 237.373737….
\]

\[
100n = 237.37
\]

\[
- n = 2.37
\]

\[
\frac{99n}{99} = \frac{235}{99} = n
\]

\[
n = 2.35.00
\]

44. \[
0.083 = \frac{83}{1000}
\]

45. \[
0.0042 = \frac{42}{10000} = \frac{21}{5000}
\]

46. \[
2.344444 \quad 100n = 234.444444….
\]

\[
100n = 234.4
\]

\[
-10n = 23.4
\]

\[
\frac{90n}{90} = \frac{211}{90} = n
\]

47. \[
\frac{5}{7} = \frac{19}{7}
\]

48. \[
\frac{1}{6} = \frac{25}{6}
\]
49. \(-3 \frac{3}{4} = \frac{(-3)(4)-1}{4} = -\frac{13}{4}\)
50. \(-35 \frac{3}{8} = \frac{((-35)(8))-3}{8} = -\frac{283}{8}\)

51. \(\frac{11}{5} = \frac{2 \cdot 5 + 1}{5} = 2 \frac{1}{5}\)
52. \(\frac{27}{15} = \frac{1 \cdot 15 + 12}{15} = \frac{12}{15} = \frac{4}{5}\)

53. \(-12 = \frac{(-1)(7) - 5}{7} = -1 \frac{7}{1}\)
54. \(-136 = \frac{(-27)(5) - 1}{5} = -27 \frac{1}{5}\)

55. \(\frac{1}{2} + \frac{4}{5} = \frac{1 \cdot 5 + 4 \cdot 2}{10} + \frac{8}{10} = \frac{13}{10}\)
56. \(\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{3}{2} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}\)

57. \(\frac{1}{6} + \frac{5}{4} = \frac{1 \cdot 2 + 5 \cdot 3}{12} + \frac{15}{12} = \frac{17}{12}\)
58. \(\frac{4}{5} \cdot \frac{15}{16} = \frac{60}{80} = \frac{3}{4}\)

59. \(\frac{5}{9} \cdot \frac{6}{7} = \frac{5 \cdot 6}{9 \cdot 7} = \frac{30}{63} = \frac{10}{21}\)
60. \(\left(\frac{4}{5} + \frac{5}{7}\right) + \frac{4}{5} = \frac{28 + 25}{35} + \frac{5}{4} = \frac{53}{28}\)

61. \(\left(\frac{2}{3} \cdot \frac{1}{7}\right) + \frac{4}{7} = \frac{2}{21} + \frac{4}{7} = \frac{2}{15} = \frac{1}{6}\)
62. \(\left(\frac{1}{5} + \frac{2}{3}\right) \cdot \frac{3}{8} = \frac{3 + 10}{15} \cdot \frac{3}{8} = \frac{13}{15} \cdot \frac{3}{8} = \frac{13}{40}\)

63. \(\left(\frac{1}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{5} \cdot \frac{1}{2}\right) = \frac{2}{15} + \left(\frac{1}{5} \cdot \frac{2}{1}\right) = \frac{2}{15} + \frac{2}{5} = \frac{2}{15} + \frac{6}{15} = \frac{8}{15}\)
64. \(\left(\frac{1}{8}\right)^7 \frac{17}{4} = \left(\frac{1}{8}\right)^7 \frac{71}{4} = \frac{71}{32} = 2 \frac{3}{12}\) teaspoons

65. \(\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5 \sqrt{2}\)
66. \(\sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{100} \cdot \sqrt{2} = 10 \sqrt{2}\)

67. \(\sqrt{5} + 7 \sqrt{5} = 8 \sqrt{5}\)
68. \(\sqrt{5} - 4 \sqrt{3} = -3 \sqrt{3}\)

69. \(\sqrt{8} + 6 \sqrt{2} = 2 \sqrt{2} + 6 \sqrt{2} = 8 \sqrt{2}\)
70. \(\sqrt{5} - 7 \sqrt{27} = \sqrt{5} - 21 \sqrt{3} = -20 \sqrt{3}\)

71. \(\sqrt{75} + 27 = 5 \sqrt{3} + 3 \sqrt{3} = 8 \sqrt{3}\)
72. \(\sqrt{5} \cdot \sqrt{6} = \sqrt{30} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3 \sqrt{2}\)

73. \(\sqrt{8} \cdot \sqrt{6} = \sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4 \sqrt{3}\)
74. \(\sqrt{\frac{18}{2}} = \sqrt{9} = 3\)

75. \(\sqrt{\frac{56}{2}} = \sqrt{\frac{28}{2}} = 2 \sqrt{7}\)
76. \(\frac{4 \sqrt{3}}{\sqrt{3}} \cdot \sqrt{3} = \frac{4 \sqrt{3}}{3}\)

77. \(\sqrt{\frac{\sqrt{5}}{\sqrt{5}}} = \sqrt{\frac{15}{5}}\)
78. \(3(2 + \sqrt{7}) = 6 + 3 \sqrt{7}\)

79. \(\sqrt{3} (4 + \sqrt{6}) = 4 \sqrt{3} + \sqrt{18} = 4 \sqrt{3} + 3 \sqrt{2}\)
80. \(\sqrt{3} (\sqrt{6} + 15) = \sqrt{18} + 45 = 3 \sqrt{2} + 3 \sqrt{5}\)

81. \(x + 2 = 2 + x\) Commutative property of addition
82. \(5 - m = m - 5\) Commutative property of multiplication
83. Associative property of addition
85. Commutative property of addition
87. Associative property of multiplication
89. Distributive property
91. Natural numbers – closed for addition $3 + 4 = 7$
93. Not closed; $\frac{1}{2}$ is not an integer
95. Not closed; $\sqrt{2} \times \sqrt{2} = 2$ is not irrational

97. $3^2 = 3 \cdot 3 = 9$
98. $3 \cdot 2 = \frac{1}{3} \cdot 3 = \frac{1}{9}$
99. $\frac{9^5}{9^3} = 9^{5-3} = 9^2 = 81$

100. $(2^2)^2 = 2^{2 \cdot 2} = 2^4 = 16$
104. $(3^2)^2 = 3^{2 \cdot 2} = 3^4 = 81$
105. $230,000 = 2.3 \times 10^5$

106. $0.000158 = 1.58 \times 10^{-3}$
107. $0.00275 = 2.75 \times 10^{-3}$
108. $4,950,000 = 4.95 \times 10^6$

112. $1 \times 10^5 = 100,000$
113. a) $(7 \times 10^3)(2 \times 10^{-5}) = 1.4 \times 10^{-1}$
114. a) $(4 \times 10^3)(2.5 \times 10^2) = 10.0 \times 10^4 = 1.0 \times 10^5$

115. $\frac{8.4 \times 10^3}{4 \times 10^2} = \frac{8.4}{4} \times \frac{10^3}{10^2} = 2.1 \times 10^1$
116. $\frac{1.5 \times 10^{-3}}{5 \times 10^{-4}} = \frac{1.5}{5} \times \frac{10^{-3}}{10^{-4}} = 0.3 \times 10^1 = 3.0 \times 10^0$

117. a) $(4,000,000)(2,000) = (4 \times 10^6)(2 \times 10^3)$
\hspace{1cm} = (4)(2) \times 10^9 \cdot 10^3 = 8.0 \times 10^9$
\hspace{1cm} b) $8.0 \times 10^9$
\hspace{1cm} b) $8.0 \times 10^9$

118. a) $(35,000)(0.00002) = (3.5 \times 10^4)(2.0 \times 10^{-5})$
\hspace{1cm} = (3.5)(2) \times 10^9 \cdot 10^{-5} = 7.0 \times 10^{-1} = 0.7$
\hspace{1cm} b) $7.0 \times 10^{-1}$

119. $\frac{96,000,000}{3,000} = \frac{9.6 \times 10^6}{3 \times 10^3} = 3.2 \times 10^3 = 3200$
120. $\frac{2 \times 10^{-6}}{4 \times 10^{-7}} = \frac{0.5 \times 10^1}{5.0}$

121. $\frac{1.49 \times 10^{11}}{3.84 \times 10^8} = \frac{3880208333 \times 10^3}{388} \approx 10^5$
122. $\frac{20,000,000}{3.60} = \frac{2.0 \times 10^7}{3.6 \times 10^5} \approx 0.55556 \times \frac{10^4}{10^5} = 0.55556$

123. Arithmetic 14, 17
125. Arithmetic $-15, -18$
127. Arithmetic 16, 19

129. 3, 7, 11, 15, $a_4 = 15$
130. $-4, -10, -14, -18, -22, -26, -30, -34$, $a_8 = -34$

131. $-20, -15, -10, -5, 0, 5, 10, 15, 20$, $a_{10} = 25$
132. 3, 6, 12, 24, 48, $a_4 = 48$
133. \( a_5 = 4(1/2)^{5-1} = 4(1/2)^4 = 4(1/16) = 1/4 \)

134. \( a_4 = -6(2)^{4-1} = -6(2)^3 = -6(8) = -48 \)

135. \( s_{30} = \frac{30(2+89)}{2} = (15)(91) = 1365 \)

136. \( s_8 = \frac{8(-4+(-2\frac{1}{2}))}{2} = \frac{(8)(-6\frac{1}{2})}{2} = -25 \)

137. \( s_8 = \frac{8(100+58)}{2} = \frac{(8)(158)}{2} = 632 \)

138. \( s_{20} = \frac{20(0.5+5.25)}{2} = \frac{(20)(5.75)}{2} = 57.5 \)

139. \( s_3 = \frac{5(1-3^4)}{1-3} = \frac{(5)(1-81)}{-2} = \frac{(5)(-80)}{-2} = 200 \)

140. \( s_4 = \frac{2(1-3^4)}{1-3} = \frac{(2)(1-81)}{-2} = \frac{(2)(-80)}{-2} = 80 \)

141. \( s_5 = \frac{3(1-(2)^5)}{1-(2)} = \frac{(3)(1+32)}{3} = \frac{(3)(33)}{3} = 33 \)

142. \( s_6 = \frac{1(1-(2)^6)}{1-(2)} = \frac{(1)(1-64)}{3} = \frac{(1)(-63)}{3} = -21 \)

143. Arithmetic: \( a_n = -3n + 10 \)

144. Arithmetic: \( a_n = 3 + (n - 1)3 = 3n \)

145. Arithmetic: \( a_n = -(3/2)n + (11/2) \)

146. Geometric: \( a_n = 3(2)^{n-1} \)

147. Geometric: \( a_n = 2(-1)^{n-1} \)

148. Geometric: \( a_n = 5(1/3)^{n-1} \)

149. Yes; 13, 21

150. Yes; 17, 28

151. No; 1, 4, 3, -1, -4, -5

152. No

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**Chapter Test**

1. 38,610 is divisible by: 2, 3, 5, 6, 9, 10

2. 2 \( \underline{840} \)
   
   2 \( \underline{420} \)
   
   2 \( \underline{210} \)
   
   5 \( \underline{105} \)
   
   3 \( \underline{35} \)
   
   7

   \( 840 = 2 \cdot 3 \cdot 5 \cdot 7 \)

3. \( [(-6) + (-9)] + 8 = -15 + 8 = -7 \)

4. \( -7 - 13 = -20 \)

5. \( [(-70)(-5)] + (8 - 10) = 350 + [8 + (-10)] = 350 + (-2) = -175 \)

6. \( \frac{4}{8} = \frac{(8)(4) + 5}{8} = \frac{32 + 5}{8} = \frac{37}{8} \)

7. \( \frac{176}{9} = \frac{(19)(9) + 5}{9} = 19\frac{5}{9} \)

8. \( \frac{5}{8} = 0.625 \)

9. \( 6.45 = \frac{645}{100} = \frac{129}{20} \)

10. \( \frac{5}{16} + \frac{4 \cdot 1}{5 \cdot 2} = \left( \frac{5}{16} \cdot 3 \right) + \frac{4}{10} = \frac{5}{48} + \frac{4}{480} = \frac{192}{480} = \frac{242}{480} = \frac{141}{240} \)
11. \( \frac{11}{12} - \frac{3}{8} = \left( \frac{11}{12} \right) \left( \frac{2}{2} \right) - \left( \frac{3}{8} \right) \left( \frac{3}{3} \right) = \frac{22}{24} - \frac{9}{24} = \frac{13}{24} \)

12. \( \sqrt{75} + \sqrt{48} = \sqrt{25 \cdot 3} + \sqrt{16 \cdot 3} = 5 \sqrt{3} + 4 \sqrt{3} = 9 \sqrt{3} \)

13. \( \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{14}}{\sqrt{49}} = \frac{\sqrt{14}}{7} \)

14. The integers are closed under multiplication since the product of two integers is always an integer.

15. Associative property of addition

16. Distributive property

17. \( \frac{4^5}{4^2} = 4^{5-2} = 4^3 = 64 \)

18. \( 4^3 \cdot 4^2 = 4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024 \)

19. \( 3^{-4} = \frac{1}{3^4} = \frac{1}{81} \)

20. \( \frac{7.2 \times 10^6}{9.0 \times 10^{-5}} = 8.0 \times 10^{11} \)

21. \( a_n = -4n + 2 \)

22. \( \frac{11[-2 + (-32)]}{2} = \frac{11(-34)}{2} = -187 \)

23. \( a_5 = 3(3)^4 = 3^5 = 243 \)

24. \( \frac{3(1 - 4^5)}{1-4} = \frac{3(1 - 1024)}{-3} = 1023 \)

25. \( a_n = 3 \cdot (2)^{n-1} \)

26. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

**Group Projects**

1. In this exercise, you may obtain different answers depending upon how you work the problem.

1) a) 2 servings  Rice: 2/3 cup, Salt: 1/4 tsp., Butter: 1 tsp.
    b) 1 serving  Rice: 1/3 cup, Salt: 1/8 tsp., Butter: 1/2 tsp.
    c) 29 servings  Rice: 5 cup, Salt: 1 7/8 tsp., Butter: 7 1/2 tsp.

2. a) Area of triangle 1 = \( A_1 = \frac{1}{2}bh = \frac{1}{2}(5)(2\sqrt{5}) = 5\sqrt{5} \)

Area of triangle 2 = \( A_2 = \frac{1}{2}bh = \frac{1}{2}(5)(2\sqrt{5}) = 5\sqrt{5} \)

Area of rectangle = \( A_r = bh = (10)(2\sqrt{5}) = 20\sqrt{5} \)

b) Area of trapezoid = \( A_t = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(2\sqrt{5})(10 + 21) = 31\sqrt{5} \)

c) Yes, same
3. Co-pay for prescriptions = 50%  Co-pay for office visits = $10  Co-pay for medical tests = 20%
   01/10:  $10 + .50 (44) = $32.00
   02/27:  $10 + .20 (348) = $47.60
   04/19:  $10 + .20 (348) + .50 (76) = $117.60
   a) Total = $197.20
   b) .50 (44) + .80 (188) + .80 (348) + .50 (76) = $488.80
   c) $500.00 – 197.20 = $302.80

4. a) 1 branch  b) 8 branches  c) 512 branches  d) Yes